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ABSTRACT

This study investigated the effects of using two microcomputer games to help children in grades 4 to 6 develop and apply concepts of probability. To achieve the object of the games, a player should choose moves ("captures") so that the probability of recapture by an opponent is minimized. Questions addressed included whether students develop understanding of this application of probability concepts as a result of experience playing, whether they develop this understanding as a result of discussing the games with an adult, and whether transfer occurs from one game version to another. Data were collected from interviews conducted with the children as they played the games. Among the data analyses were measures of the children's use of four known strategies and measures of the congruence between their statements about their srategies and their actual play. Results indicate that some subjects had little difficulty applying probability concepts and explaining their strategies; others recognized that some positions changed hands more than others, but did not seem able to relate this to probability. Based on these and other results, it was concluded that the games are suitable activities and that discussion of them is valuable. (Author/JN)

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AN ASSESSMENT OF ELEMENTARY SCHOOL STUDENTS' DEVELOPMENT AND APPLICATION OF PROBABILITY CONCEPTS WHILE PLAYING AND DISCUSSING TWO STRATEGY GAMES ON A MICROCOMPUTER

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Submitted to the faculty of the Graduate School
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AN ASSESSMENT OF ELEMENTARY SCHOOL STUDENTS' DEVELOPMENT AND APPLICATION OF PROBABILITY CONCEPTS WHILE PLAYING AND DISCUSSING TWO STRATEGY GAMES ON A MICROCOMPUTER

Thomas Leonard Schroeder, Ph. D.
Indiana University, 1983

Chairperson: Dr. John F. Le Blanc

To achieve the object of the games used in this study, a player should choose moves ("captures") so that the probability of recapture by the opponent is minimized. The questions of the study included whether students develop understanding of this application of probability concepts as a result of experience playing, whether they develop this understanding as a result of discussing the games with an adult, and whether transfer occurs from one game version to another.

The methodology of a "teaching experiment in the experiencing mode" was adopted. Audiotaped interviews were conducted with children in grades four to six as they played the games. Quantitative data were collected by routines in the computer programs; qualitative data were collected from the children's verbal reports. Data analyses included measures of the children's use of four known strategies (conscious and consistent rules for choosing moves), measures of the congruence between their statements about their strategies and their actual play, qualitative assessment of their statements reflecting

understanding and application of probability concepts, and estimates of the transfer of concepts and strategies from one game version to the other.

Some subjects had little difficulty applying probability concepts and explaining their strategies. Others recognized that some positions changed hands more than others, but did not seem able to relate this to probability. Most students had difficulty producing exhaustive lists of the ways positions could be recaptured, and they made some errors in implementing their strategies.

It was concluded that the games are suitable activities, that discussion of them is valuable, and that the study's methodology was appropriate and fruitful.

Approved and accepted by John F. LeBlanc ...., Chairperson

Frank K. Lester Jr. ....

Maynard Thompson ....

Jerry A. McIntosh ....

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— T. L. S.

Table of Contents

Chapter Page

1 INTRODUCTION 1

A Point of View 2

The Setting and Significance of the Problem 5

Games 5

Probability 8

Computers 10

The Statement of the Problem 12

Definitions of Terms 13

Limitations 25

2 RELATED RESEARCH 28

Children's Probability in the Absence of Instruction 28

Piagetian Studies 29

The Work of Fischbein 41

Other Investigations 44

Instruction in Probability 48

Games and Computer Gaming 52

Summary 59

Chapter Page

3 METHODOLOGY 61

General Considerations 61

The Plan of the Study 70

Quantitative Data -- Collection and Analysis 71

Qualitative Data -- Collection and Analysis 82

Selection of Subjects 87

4 FINDINGS 91

Experimental Observations 91

General Observations 91

Alan 93

Beth 97

Carl 100

Dawn 107

Elsa 111

Fred 115

Observations in Relation to the Major Questions of the Study 122

Question 1 123

Question 2 125

Question 3 126

Question 4 128

Question 5 131

Question 6 133

Question 7 136

Chapter Page

5 DISCUSSION 138

Summary 138

Conclusions 140

Implications for Further Research 144

Implications for the Content of Further Research 144

Implications for the Methodology of Further Research 148

Implications for Practice 150

REFERENCES 155

APPENDICES 160

Appendix 1: Forms Used in Obtaining Consent 160

Appendix 2: Quantitative Data Summary 163

VITA 182

List of Tables

Table	Description	Page(s)
3.1	Definition of strategy 1, avoid self capture, and variable S1	75
3.2	Definition of strategy 2, attempt opponent capture, and variable S2	76
3.3	Probability that position P can be recaptured by the opponent given that N turns remain to be played in Capture 1	77
3.4	Probability that position P can be recaptured by the opponent given that N turns remain to be played in Capture 2	78
3.5	Definition of strategy 3, choose minimum ways, and variable S3	80
3.6	Definition of strategy 4, coordinated strategy, and variable S4	83
3.7	Potential interview questions and comments on their use and purpose	85-86
3.8	Name, sex, age, and grade of subjects	90

List of Figures

Figure	Description	Page(s)
1.1	Rules for Capture 1	14-15
1.2	Rules for Capture 2	16-18
1.3	Typical screen displays when playing Capture 1	20-21
1.4	Typical screen displays when playing Capture 2	22-24
3.1	Classification of Soviet research methodologies	65
3.2	Number of dice roll outcomes which permit the capture of each board position in Capture 1 and Capture 2	74
3.3	Definition of the function used to define strategy 4, coordinated strategy, in Capture 1 and Capture 2	81

CHAPTER 1

Do not, then, my friend, keep children
to their studies by compulsion, but by play.

-- Plato, The Republic

This exhortation from ancient Greece gives an indication of the long history of the use of games in education. Earlier in the present century Dewey (1928) advocated the inclusion of games as an integral part of the school curriculum, and in the last two decades alone there has been an explosion both in the supply of and in the use of educational games of many types, designed to serve a variety of purposes, and adapted to a wide range of educational settings. In mathematics teaching games are sometimes used primarily to provide motivation, to improve attitudes, or to disguise computational practice activities. At other times games are used to embody concepts or to give first-hand experience with the operation of systems which function under the constraint of rules. The games under investigation in this study have been designed both to provide practice with arithmetic skills as well as, and more importantly, to promote the development and application of concepts of probability. In order to put the purposes and methods of using these games into perspective, it may be useful to examine in some detail an educational point of view and a conceptualization of the nature of intelligence and learning on which they have been based.

A Point of View

A fruitful stance to adopt in approaching important issues in education is that of the epistemologist and, in particular, the genetic epistemologist. Whether one chooses to view the task of teaching as the transmission of knowledge to the student or as its nurture in the student, educators are concerned with the nature of knowledge and how knowledge develops. Writers who have taken an approach along these lines include Richard Skemp in Intelligence, Learning and Action (1980), Zoltan Dienes in a number of works, such as, for example, Concept Formation and Personality (1965), Seymour Papert in Mindstorms (1980), and, of course, Jean Piaget the foremost to call himself a genetic epistemologist.

Genetic epistoemolgists may also be called constructivists since they believe that knowledge is not so much transmitted as it is constructed by the active (cognitive) effort of the individual in representing and abstracting from experience mental schemas, that is structures of understandings about the world. For them, an individual's experience both in school and in everyday life outside of school is constantly being assimilated into cognitive structures, and cognitive structures are undergoing change over time to accommodate experiences which do not fit previously developed schemas.

Educators who subscribe to this point of view recognize the child as an epistemologist in his or her own right and consider education's task to be one of providing experiences that are compatible with the learner's cognitive structures and that will enrich and extend and eventually change those structures to include more knowledge of the world and more powerful schemas for understanding it. Piaget describes certain methods of preschool education of which he approves as being characterized by "an appeal to the spontaneous activity of the child itself in the direction of a cognitive organization that prepares for the mental operations normally appearing at about age seven to eight." (1973, p. 6) He advocates "the use of active methods which give broad scope to the spontaneous research of the child or adolescent and require that every new truth to be learned be rediscovered or at least reconstructed by the student and not simply imparted to him." (1973, p. 16) He describes a "truly modern method for teaching the new mathematics" as one that "would consist in speaking to the child in his own language before imposing on him another ready-made and over-abstract one, and, above all, in inducing him to rediscover as much as he can rather than simply making him listen and repeat." (1973, p. 19)

The mathematics educator Dienes is well known for his applications of the constructivist point of view. Dienes' understanding of the nature of learning has led him to propose a system for learning mathematics that has six stages: (1) free play, (2) games, (3) searching for commonalities, (4) representation, (5) symbolization, and (6) formalization (Dienes & Golding, 1971). In stage (1), free play, the learner explores an object or an apparatus just to see what can be done with it, but this stage soon gives way to manipulation under rule-bound conditions, structured play, and games, stage (2). In stage (3), searching for commonalities, the learner begins to go beyond the particular attributes of concrete situations and looks for and uses language to describe regularities and relationships among the objects and the rules that have been manipulated in stages (1) and (2). This leads into first naturalistic representation, stage (4), then symbolization, stage (5), and finally formalization, stage (6), of the structures that were experienced, manipulated, and discussed informally at earlier stages. Two important ingredients in this process are the particular manipulative materials and structured games that are used. Often these are quite specialized. For example, Dienes' well known multi-base arithmetic blocks are a series of wooden blocks the specific shapes and sizes and markings of which embody the structure 1, b, b^2, b^3, ... that is used in numeration in base b (for b = 2, 3, ... , 10). Corresponding to the structured materials are rules for fair trades among the blocks, often spontaneously discovered or invented by children and used by them in structured play.

Dienes and Jeeves (1965, 1970) and Branca and Kilpatrick (1972) and others have investigated this system as it is applied to the learning of the structures of mathematical groups by means of games that embody those structures. Dienes has also speculated that "most mathematical structures can be learned by playing skillfully contrived and excitingly motivating games of a mathematical nature" (Dienes, 1972, p. 64). The games under investigation in the present study embody the mathematical structure of a probability experiment. Their rules were contrived by the investigator to force children to make decisions with the expectation that in doing so the children would begin to search for relationships between the structures of the game situations and the structures of the arithmetic operations on pairs of whole numbers with which they are familiar.

Much more will be said in Chapter 3, which describes in detail the games and their development, about the particular structures that are involved in this study. More will also be said about the nature and role of the discussion between child and adult that is a vehicle for the search for relationships and commonalities. Suffice it at this point to describe the point of view regarding learning and the nature of knowledge that has been taken in designing this study.

The Setting and Significance of the Problem

Three important facets of this study are (1) that it is concerned with two games and a method of using them in teaching, (2) that the games have been designed to promote children's development and application of concepts of probability, and (3) that the games are played on a microcomputer. These facets are in no sense orthogonal; on the contrary, the teaching and learning of probability suggests the use of games, and the development of and academic study of games has been spurred by developments in microcomputer technology. Yet in describing the setting of the problems under investigation in this study it is useful to consider each of games, probability, and microcomputers in turn.

Games

The body of published research on the use of games in education is large and steadily growing. In it can be found information on effects of many sorts, both cognitive and affective, of a wide variety of games, versions of games, and adaptations of games, used in diverse educational settings, and serving a broad range of educational roles and purposes. Because this research is highly multi-dimensional, and because there has been little consistency in the identification and manipulation of variables related to game playing situations, it is not possible to state proven generalizations about relationships among characteristics of games, players, and educational outcomes. Nonetheless several important insights can be gained from a study of this research.

In most cases researchers have reported that students found game playing to be an enjoyable activity, and in some cases there is evidence that the enjoyment spilled over, causing positive changes in students' attitudes toward school subjects. In the few instances of negative affective results, the researchers have usually identified causes, such as debilitating anxiety in some individuals resulting from pressure to compete. (See, for example, Bonner, 1974.) These findings suggest that game playing can be expected to be a positive experience, but that care must be exercised in the selection of appropriate games and in the management of competitive situations.

Much of the research on games has focused on the effectiveness of games designed to provide practice of skills. Most often the games under study have been found to be as effective as or more effective than alternative treatments in achieving mastery of the skills in question. In one case (Weusi-Puryear, 1975), a game treatment was found to be superior even though subjects using the game performed fewer repetitions of the skill than did subjects using an alternative drill and practice format.

Other research, more closely related to the concerns of the present study, has dealt with the use of games to promote higher order cognitive abilities such as understanding, application, and problem solving. The research, previously mentioned, related to the learning of the structures of mathematical groups by means of games embodying those structures exemplifies the use of games in developing understanding. Frietag (1974) studied the use of games to develop understanding and application of mathematical concepts. Others including Goldberg (1980), Kraus (1980), and Fluck (1982) have used strategy games, that is games in which the player must discover or learn (possibly from formalized instruction) methods for determining the moves a player should make in order to be more likely (perhaps even certain) to win. These studies, which are reviewed in greater detail in Chapter 2, suggest that there may be transfer from playing strategy games to more general problem solving situations, especially if the similarities between the processes of developing strategies for games and the processes for finding solutions to problems are pointed out to students. It may be concluded that the playing of games with discussion focusing on strategies for playing may be effective in promoting understanding of concepts and their application in a problem solving task, namely the construction of strategies for games. The present investigation is intended to determine whether particular games and discussion methods can achieve these goals.

Probability

The study of probability had its origins, historically, in the study of gambling and games of chance. Although probability concepts and results are now applied to statistics, economics, meteorology, genetics, and many other fields, games that involve rolling dice, spinning spinners, or drawing cards are still used as elementary embodiments of concepts related to probability. By playing and discussing a game, notions such as sample space, outcomes, events, estimated or compared probabilities, and equal and unequal likelihoods can be dealt with in concrete terms. Strategy games like the ones under investigation in this study provide a setting for studying children's application of these concepts as they construct, describe, and evaluate their strategies, that is their conscious and consistent rules or procedures for selecting moves in the games.

The status of probability as a topic in the elementary school curriculum is ambiguous. On the one hand, several instances of pressure from professional organizations of mathematics educators to give probability greater prominence can be identified. Some examples include the National Council of Supervisors of Mathematics' "Position Paper on Basic Skills" (NCSM, 1976), a series of articles in the *Arithmetic Teacher* (Choate, 1979; Enman, 1979; Jones, 1979), and especially the National Council of Teachers of Mathematics' 1981 yearbook, *Teaching Statistics and Probability* (Shulte, 1981). Yet despite strong support for probability on the part of leading mathematics educators, a review of curriculum guides and textbooks actually in use suggests that work with concepts of probability does not constitute a large part of the mathematics curriculum of elementary schools in North America. Perhaps one reason for this is that elementary school teachers, who tend to be generalists rather than specialists in a single subject such as mathematics, have limited background with the content of and the teaching of this non-traditional topic. Whatever may be the reasons for the gap between proposals and practice, it is clear that an effective program to promote the teaching of probability in elementary schools will require substantial research, development, and dissemination efforts. These would include further development of the rationale for teaching probability, continued study of the feasibility of teaching probability, and expanded development and dissemination of materials both for students and for teachers. Studies such as the present one may contribute to this process in at least two ways: (1) by shedding light on the processes by which children develop and apply certain probability concepts, and (2) by assessing the effects of a methodology and particular student activities designed to promote students' abilities to understand and apply these concepts.

Research related to the origins and development of probability ideas in children, both in the presence of and in the absence of formal instruction, is reviewed in Chapter 2. That research suggests that children and adolescents develop some understandings of chance and probability without the benefit of any formal instruction. Analyses of the characteristics of children's responses in interviews and tests confirm that there exist stages in the development of children's ideas about probability that correspond to Piagetian stages of development for other ideas such as number, speed, and causality (Piaget & Inhelder, 1951, 1976). Generally speaking, it has been shown to be feasible to introduce informally concepts of probability to children of elementary school age.

Computers

For the practitioner and the researcher alike there can be many advantages in using microcomputer technology to present games to children. Consider the mundane issue of storage, for example. A copy of a computer game which can be stored (along with several other games) on a single diskette or cassette tape might require a large boxful of boards, dice, playing pieces, score pads, and other paraphrenalia if the game were to be played with actual objects. Similarly, the distractions due to noise and physical motion may well be less when players play on a microcomputer's keyboard than when they roll dice, spin spinners, and act out moves. From the standpoint of learning a new game, there can be advantages to the self-paced tutorial routines that are often used to explain the rules and moves of a microcomputer game. Moreover, many computer games allow players to learn a new game by playing it against a computer-simulated opponent. In some games a player can control the pace of the learning sessions by setting the level of difficulty of the game or the degree of sophistication with which the computer-simulated player plays. In research studies, the use of a computer-simulated player gives the researcher control over the possible effects of an opponent's play on the player's play. Information about each move in a game can be collected unobtrusively, accurately, and in great detail by other routines embedded in the program.

At this point in time microcomputers are being introduced into schools, especially elementary schools, at a rate which has been viewed with alarm by some commentators and with satisfaction by others. Whether one should be glad that schools have been so quick to adopt the latest technology or worried that yet another bandwagon is developing is hard to decide. In either case, an important issue for education in the 1980's is how microcomputers should be used in schools; that they will be used seems beyond dispute. Before the "should" question can be answered, much more needs to be known about how microcomputers can be used. One prominent mathematics educator has assessed the current situation as follows.

> How we use computers in education may well shape the future of education. Though it won't be the only influence, it is likely to be an important one. This is alarming because decisions about the educational uses of computers are not being made in the thoughtful, careful way that is called for.
>
> (Davis, 1981, p. 149)

In approaching the issue thoughtfully and carefully, Davis recommends that fundamental goals and fundamental values be reexamined, and that decisions be made not on the basis of scores on paper-and-pencil tests but rather through the use of methods that reveal individual students' status and performance, through task-based interviews, and through analyses of interactions in actual computer lessons. These views of Davis have guided the selection of both the topic and the methods of the present investigation.

The Statement of the Problem

This study sets out to explore the effects of using two games together with questions about the games for the purpose of helping children in grades four to six to develop and apply concepts of probability. The games, which are played on a microcomputer, were invented by the investigator and given the name 'Capture.' Although several versions of Capture have been produced, only two versions (hereafter called Capture 1 and Capture 2) have been included in this study. In both versions a person plays against a computer-simulated opponent. In both the roll of dice determines the options a player has in choosing a position on the number line to capture. The object of the games is to capture and to hold at the end of the game more places than the opponent. The games have been designed to present children with choices with the expectation that they will recognize a need to develop and apply concepts of counting the outcomes of probability experiments and estimating and comparing probabilities of events as the basis for making their choices. The most important characteristic of this study is its focus on children's thinking as it develops from and is applied to the playing of the games.

The specific questions to be answered by the investigation are

(1) What strategies do children in grades four to six spontaneously use in playing Capture?

(2) Do children's strategies for playing Capture change as they gain experience playing the game? If so, how?

(3) Do children's strategies for playing Capture change as a

result of being asked by an adult questions about their play? If so, how and as consequences of what specific questions?

(4) How do children describe, justify, and explain their strategies?

(5) Are children's moves in playing Capture consistent with the strategies they claim to be using to determine their moves?

(6) To what extent do children use mathematical concepts (especially concepts of probability and counting) in determining and explaining their strategies for playing Capture?

(7) To what extent do children generalize or transfer their strategies for playing Capture and the mathematical concepts upon which their strategies are based from one version of Capture to another?

Definitions of Terms

Several terms used in this and later chapters have precise meanings as set out below.

The names 'Capture 1' and 'Capture 2' refer to two games which are played on an Apple II Plus microcomputer system. Figure 1.1 and Figure 1.2 respectively suggest operational definitions of these terms by giving the rules of the games as they are presented by the computer. When the rules are being presented, the video screen display includes the heading 'CAPTURE 1' or 'CAPTURE 2' followed by the appropriate frame as shown in Figure 1.1 or Figure 1.2, and by the message 'PRESS ANY KEY TO GO ON.' at the bottom of the screen. The player controls the pace with which the rules are presented by pressing a key to move to the next frame. After all the rules have been presented, the player is offered the opportunity to have all the rules presented again from the beginning. In both versions of Capture used in this study player 1 is computer-simulated. For this reason

Figure 1.1: Rules for Capture 1

RULE 1:	TWO PLAYERS PLAY ON A NUMBER LINE THAT GOES FROM 0 TO 36. EACH PLAYER HAS A MARK -- EITHER X OR O.
RULE 2:	THE PLAYERS TAKE TURNS. EACH PLAYER GETS 15 TURNS. ON EACH TURN THE APPLE WILL ROLL TWO DICE MARKED WITH THE NUMBERS FROM 1 TO 6.
RULE 3:	YOU CAN USE THE NUMBERS ON THE DICE TO CAPTURE ONE PLACE ON THE NUMBER LINE FROM 0 TO 36.
RULE 4:	YOU CAN CAPTURE THE PLACE THAT IS THE SUM, OR THE DIFFERENCE, OR THE PRODUCT OF THE TWO NUMBERS. YOU CAN ALSO USE THE NUMBERS AS TENS AND ONES.

Figure 1.1 (Continued)

FOR EXAMPLE, IF YOU ROLLED 3 AND 2, YOU COULD CAPTURE:

3 + 2 = 5 OR
3 - 2 = 1 OR
3 X 2 = 6 OR
3 TENS AND 2 ONES = 32 OR
2 TENS AND 3 ONES = 23.

RULE 5: YOU CAN CAPTURE ANY PLACE THAT CAN BE MADE WITH THE DICE. YOU CAN EVEN CAPTURE A PLACE THAT WAS CAPTURED BEFORE BY YOU OR BY YOUR OPPONENT.

RULE 6: THE OBJECT OF THE GAME IS TO CAPTURE AS MANY PLACES AS YOU CAN. THE WINNER IS THE PLAYER WHO HAS THE MOST PLACES AT THE END OF THE GAME. THE GAME IS OVER WHEN BOTH PLAYERS HAVE TAKEN 15 TURNS.

Figure 1.2: Rules for Capture 2

RULE 1:	TWO PLAYERS PLAY ON A NUMBER LINE THAT GOES FROM 0 TO 36. EACH PLAYER HAS A MARK -- EITHER X OR O.
RULE 2:	THE PLAYERS TAKE TURNS. EACH PLAYER GETS 15 TURNS.
RULE 3:	ON EACH TURN THE APPLE WILL ROLL TWO DICE. ONE IS MARKED WITH THE NUMBERS FROM 1 TO 6. THE OTHER IS MARKED WITH THE OPERATIONS +, -, AND X.
RULE 4:	YOU CAN CAPTURE ONE PLACE ON THE NUMBER LINE. THE PLACE YOU CAPTURE MUST BE MADE WITH THE NUMBER AND OPERATION ON THE DICE AND ANY ONE-DIGIT NUMBER THAT YOU CHOOSE.

Figure 1.2 (Continued)

HERE ARE SOME EXAMPLES.

IF YOU ROLLED 3 AND X, YOU COULD CAPTURE 3 X 0 = 0, OR 3 X 1 = 3, AND SO ON UP TO 3 X 9 = 27.

YOU COULD ALSO CAPTURE 0 X 3 = 0, OR 1 X 3 = 3, OR 2 X 3 = 6, AND SO ON.

IF YOU ROLLED 3 AND -, YOU COULD CAPTURE 3 - 0 = 3, OR 3 - 1 = 2, OR 3 - 2 = 1, OR 3 - 3 = 0.

YOU COULD ALSO CAPTURE 9 - 3 = 6, OR 8 - 3 = 5, OR 7 - 3 = 4, AND SO ON.

RULE 5: YOU CAN CAPTURE ANY PLACE THAT CAN BE MADE WITH THE DICE. YOU ON EVEN CAPTURE A PLACE THAT WAS CAPTURED BEFORE BY YOU OR BY YOUR OPPONENT.

Figure 1.2 (Continued)

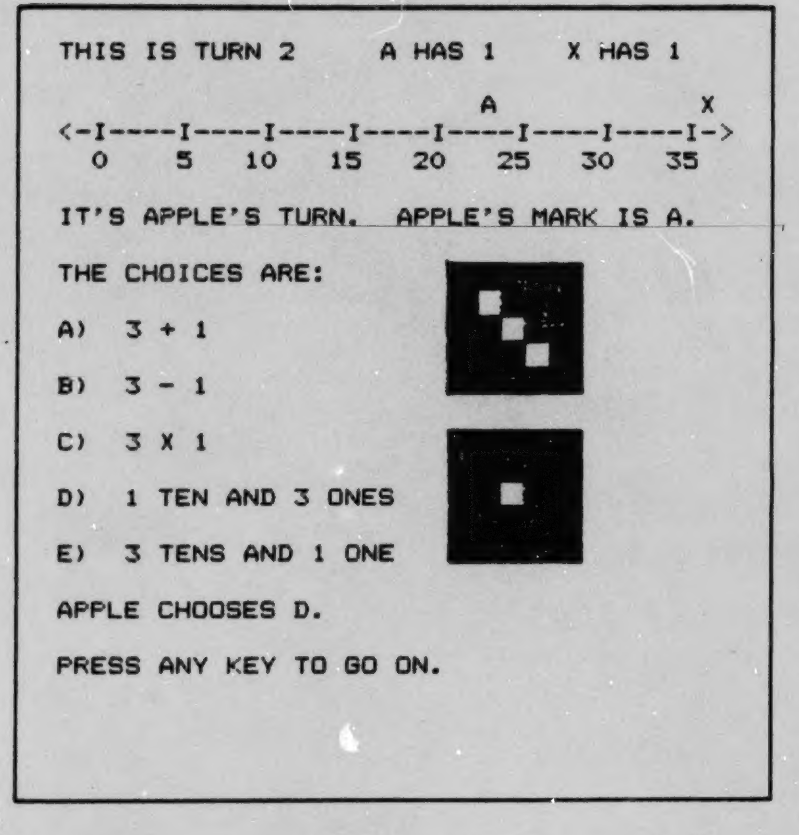
RULE 6: THE OBJECT OF THE GAME IS
TO CAPTURE AS MANY PLACES
AS YOU CAN. THE WINNER IS THE
PLAYER WHO HAS THE MOST PLACES
AT THE END OF THE GAME.
THE GAME IS OVER WHEN BOTH
PLAYERS HAVE TAKEN 15 TURNS.

player 1 is referred to as 'Apple' and player 1's mark is 'A;' player 2's mark is 'X.' Figures 1.3 and 1.4 elaborate the operational definitions of Capture 1 and Capture 2 respectively by showing typical video screen displays while playing.

The phrase 'strategy for playing Capture' is used extensively. A 'strategy' is a conscious and consistent procedure, rule, or set of procedures or rules used by a player to choose his or her moves. When players claim to have no particular reason for choosing any one of the possible moves, it may be said that their strategy is to choose by whim. More often players do have reasons for choosing their moves, and those reasons expressed in general terms constitute their strategies. Four distinct strategies for playing Capture are described in detail in Chapter 3. These strategies can be shown to have value in increasing the probability of winning. Other strategies, which may or may not increase the probability of winning, are also possible.

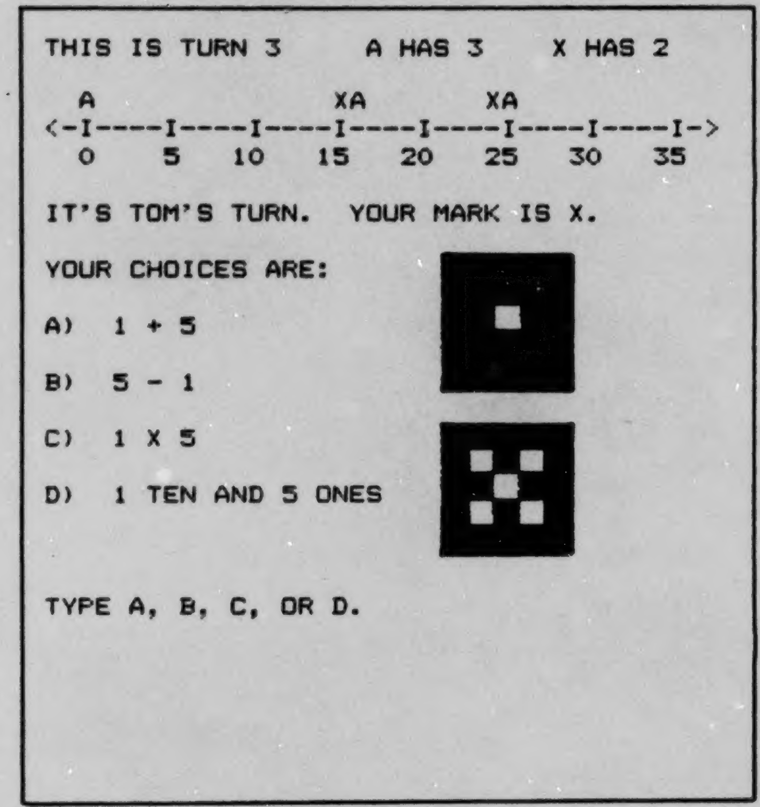
This study is concerned with children's development and application of concepts of probability and counting. In general, 'concepts of probability' are concepts involving the relative frequencies, estimated or computed, predicted or observed, of events in the sample space of a probability experiment. The distinction can be made between 'experimental probability' and 'theoretical probability.' Experimental probability is derived from experience with a probability experiment and the estimation or computation of relative frequencies of events. Experimental probability may also be called 'empirical,' 'observed,' or 'actual' probability. Theoretical

Figure 1.3: Typical screen displays when playing Capture 1



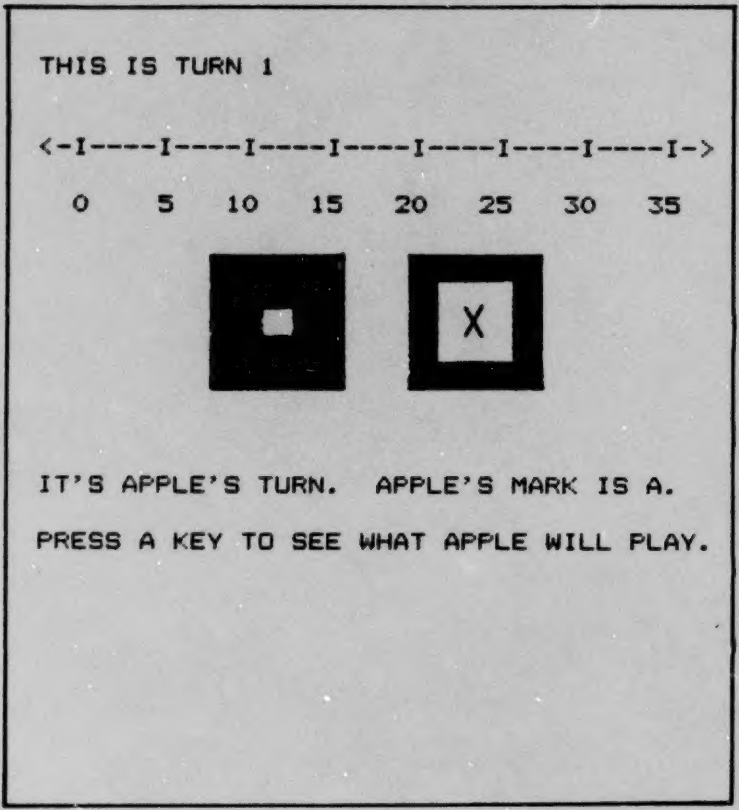
```
THIS IS TURN 2      A HAS 1     X HAS 1

                            A            X
<-I----I----I----I----I----I----I----I->
  0    5   10   15   20   25   30   35

IT'S APPLE'S TURN.  APPLE'S MARK IS A.

THE CHOICES ARE:

A)  3 + 1

B)  3 - 1

C)  3 X 1

D)  1 TEN AND 3 ONES

E)  3 TENS AND 1 ONE

APPLE CHOOSES D.

PRESS ANY KEY TO GO ON.
```

Figure 1.3 (Continued)



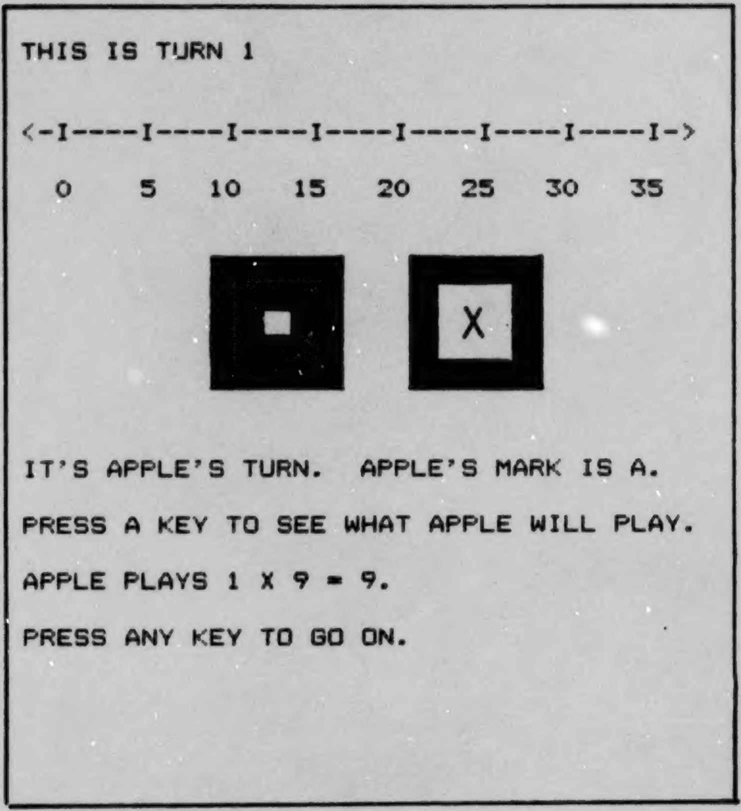
```
THIS IS TURN 3      A HAS 3    X HAS 2

  A               XA        XA
<-I----I----I----I----I----I----I----I->
  0    5   10   15   20   25   30   35

IT'S TOM'S TURN.  YOUR MARK IS X.

YOUR CHOICES ARE:

A)  1 + 5

B)  5 - 1

C)  1 X 5

D)  1 TEN AND 5 ONES


TYPE A, B, C, OR D.
```

Figure 1.4: Typical screen displays when playing Capture 2



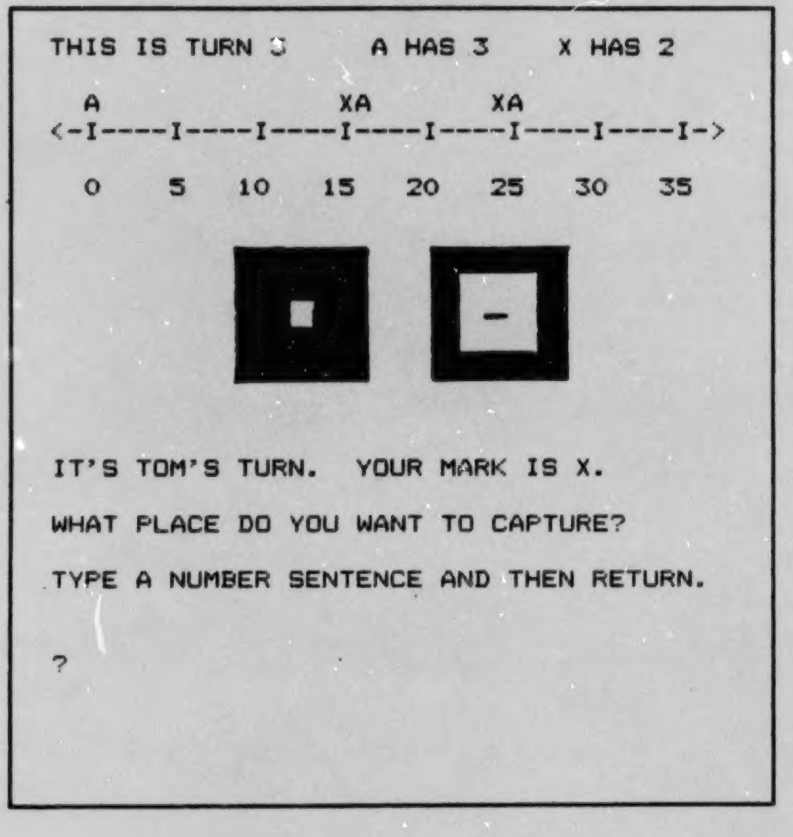
```
THIS IS TURN 1

<-I----I----I----I----I----I----I----I->

  0    5   10   15   20   25   30   35

                              X

IT'S APPLE'S TURN.  APPLE'S MARK IS A.

PRESS A KEY TO SEE WHAT APPLE WILL PLAY.
```

Figure 1.4 (Continued)



```
THIS IS TURN 1

<-I----I----I----I----I----I----I----I->

  0    5   10   15   20   25   30   35

IT'S APPLE'S TURN.  APPLE'S MARK IS A.

PRESS A KEY TO SEE WHAT APPLE WILL PLAY.

APPLE PLAYS 1 X 9 = 9.

PRESS ANY KEY TO GO ON.
```

Figure 1.7 (Continued)



```
THIS IS TURN 3      A HAS 3     X HAS 2

  A                     XA          XA
<-I----I----I----I----I----I----I----I->

  0    5   10   15   20   25   30   35

IT'S TOM'S TURN.  YOUR MARK IS X.

WHAT PLACE DO YOU WANT TO CAPTURE?

TYPE A NUMBER SENTENCE AND THEN RETURN.

?
```

probability is estimated or computed on the basis of an assumption that in a fair probability experiment certain outcomes which are symmetrical ought to be equally likely. Theoretical probability may also be called 'predicted' or 'expected' probability. When events in a probability experiement are compound events (i. e. they are comprised of more than one simple outcome of the experiment), 'concepts of counting' may be applied in estimating or computing their theoretical probability.

Children may be said to 'apply concepts of probability' when they describe their strategies for choosing among possible moves in terms of the chances, likelihoods, or probabilities of a player being able to capture the different possible positions. Children may be said to 'apply concepts of counting' when they describe their strategies for choosing among possible moves in terms of the respective numbers of outcomes of rolling the dice which would permit a player to capture the different possible positions.

Strategies and/or the concepts on which strategies are based may be said to be 'generalized' or 'transfered' when there is evidence that children describe their strategies for playing Capture 2 in the same terms as they use when describing the analogous strategy for playing Capture 1, or when they refer to similarities between their strategies for playing Capture 1 and Capture 2.

Limitations

The overall goal of this study and the seven specific questions posed above have been chosen so as to be relevant to the current state of affairs in mathematics education and consistent with the point of view stated earlier in this chapter. The scope of the study has been limited to questions that are both interesting and potentially answerable with available resources. Some particularly noteworthy limitations are the following.

(1) No attempt has been made to compare the effects of playing and discussing Capture with the effects of other educational activities. The purpose of this study is only to evaluate the potential of the games and to gain insights into ways they can be used to promote the development and application of concepts in a problem solving situation.

(2) This study includes no investigation of situations in which children play and discuss Capture with their peers. While it is recognized that such arrangements can be very fruitful and that they have practical advantages when access to microcomputers is limited, this study focuses on individual children's thinking and actions. Having the children play against a computer-simulated opponent that always plays using particular strategies gives the interviews consistency and provides the opportunity to discuss regularities that the children may observe in the computer-simulated player's moves.

(3) Systematic investigation of the effects of asking specific questions in specific orders at specific times in the interviews has not been undertaken. Although lists of questions were prepared in advance with some care and attention to detail, the actual selection and timing of questions was left to the judgement of the interviewer as he assessed the dynamics of the interactions.

(4) Like all empirical research, this study is limited by the particulars of the conditions under which it has been conducted. Moreover, the number of subjects is relatively small, and the amount of time spent with each was fairly short. The ability to draw inferences from the records of the interviews is further limited by the perceptiveness and skill of the interviewer, both during the interviews and in his subsequent reflections on them.

CHAPTER 2

RELATED RESEARCH

The eduational research that provides relevant background for the present study includes studies of probability, of games, and of computers. This chapter focusses mainly on probability, since computer gaming is only the vehicle with which to approach probability. The number of published research reports having to do with children's understanding and abilities in probability is large. A convenient way to divide this literature is to consider first the studies that deal with children who have not been given formal instruction in probability, and then those that discuss the results of using various kinds of teaching activities with children. This is done in the first two sections of this chapter. The research on uses of games and computers in mathematics education is also extensive, but in these categories the research that is relevant to the present study is more limited. In the final section of this chapter certain studies in these two areas which have a direct bearing on this investigation are reviewed.

Children's Probability in the Absence of Instruction

A number of researchers have investigated the development of children's understanding of ideas related to chance. The pioneering work in this area was done by Piaget, Inhelder, and their colleagues; their results were first published (in French) in book form in 1951.

Another major contribution to this field is a book by Fischbein published in 1975. Many other investigators have produced journal articles and doctoral dissertations dealing with this topic, as well.

Piagetian Studies

For mathematics educators the work of Piaget and his colleagues often serves as a backdrop providing a language and a context within which to discuss the nature of children's knowledge and the characteristics of their cognitive functioning and development. There exist Piagetian studies dealing with a broad range of topics relevant to mathematics education, including number, space, time, motion and speed, logic, causality, and chance, to name a few. The importance of Piaget's work derives from two complementary facts: first that empirical clinical investigations into children's thinking across the range of content areas have supported Piaget's generalized hypotheses about children's cognitive development, and second that useful insights into children's thinking often result when Piagetian analysis is applied to particular topics.

Piaget views children's cognitive development as taking place in stages. The first, or preoperational, stage is typical of children aged four to seven years; the second, or concrete operational, stage is typical of children from seven to eleven years of age; the third, or formal operational, stage is typical of children over age 11. Piaget maintains that the order of occurrence of the stages is invariant, but the ages at which the stages appear may vary considerably both from child to child and within the same individual with respect to different contents. Transitions between stages occur through the dual processes of assimilation and accommodation, and children may be observed in transition between stages or at various points early or late in a stage. Children at different stages show qualitative differences in their reactions to tasks and questions, and it is on the basis of these differences that the characteristics of the thinking in the three stages are inferred.

A central concept in Piagetian theory is the idea of an operation. "Logical and arithmetical operations constitute systems of actions interrelated in a rigorous way and always reversible, this reversible aspect rendering deduction possible." (Piaget & Inhelder, 1976, p. 212) In the preoperational stage true (reversible) operations are lacking, so reasoning is prelogical. In the concrete operational stage operations (cognitive actions) are performed on what is actual, present, or observable; in the formal operational stage operations on the possible or the potential permit propositional, hypothetical-deductive thought. The importance of logico-mathematical operations in the Piagetian analysis of probability is summed up by the following observation: "As we have seen constantly by contrast with operations, chance is gradually discovered, and it is by constant reference to the structures of operations that chance is finally understood and yields a system of probabilities." (Piaget & Inhelder, 1976, p. 212) The following summaries of selected studies by Piaget and Inhelder (1976) of children's thinking about randomness, chance, probability, and combinatorial operations (counting) show how the understanding of these concepts develops and how this development is related to the development of concrete and formal operations.

A study of the concept of randomness used a rectangular tray resting on a pivot so that it could seesaw. Along one of the edges of the tray parallel to the pivot was placed a row of red beads and a row of white beads separated by a divider. Children were asked to predict the outcomes after tilting the tray back and forth once, several times, and a large number of times. Children's predictions of when (if ever) the beads would return to their original positions were also noted. The subjects were also asked to draw pictures of the paths of the individual beads during the process.

The preoperational children either denied progressive mixing or else believed it would take place in a regular manner, for example a certain number of the reds crossing over and exchanging positions with an equal number of whites. One child, having observed that the colors became mixed, drew a picture showing three reds then three whites on one side of the divider and six whites on the other side. Among other things this shows that the child did not understand the conservation of the numbers of red and white beads during the transformation. None of these children predicted progressively more thorough mixing, and most of them viewed the return to the original arrangement as necessary ("coming back to their places"). Piaget and Inhelder attribute these results to animistic thinking and a lack of understanding of the possible permutations.

The concrete operational children understood the idea of progressive mixing, as shown by comments such as "it can still get mixed up even more." Their drawings included irregular alternation of colors, but regularities and symmetries in the paths of the beads were still noted. The return to the original arrangement was seen as possible but improbable.

The formal operational children also understood the progressive mixing and represented it as the result of random trajectories with many intersections and collisions. The return to the original arrangement was seen as possible, but highly unlikely. These children seemed to recognize that with eight beads of each color the number of different arrangements was very large, and that with fewer beads or more tries a fortuitous return to the original arrangement would be more likely.

A second series of studies was concerned with the prediction of the shapes of probability distributions. In one type of apparatus marbles were poured through a funnel into rows of bins below, producing centered, symmetrical distributions approximating the normal curve. In another apparatus beads were shaken through a screen onto a sheet of squared paper below it, representing drops of rain falling at the beginning of a small shower, and producing a more or less uniform distribution.

Children in the preoperational stage predicted for the centered distributions various distributions including irregular, assymetric, and even discontinuous ones. The predicted distributions did, however, tend to have a maximum in the central region. For the uniform distribution, there was a tendency to predict correctly, but the children often overgeneralized, predicting that each bead would go to a different square in strict obedience to a pattern.

The children in the concrete operational stage tended to predict distributions that were both symmetrical and concentrated in the central regions when working with the first apparatus. They did not, however, recognize that the distributions would tend to become more regular and more symmetrical as the number of marbles dropped increased. In predicting the results of using the uniform distribution apparatus they were generally correct, and they did not exhibit the artificial regularities common among the preoperational children.

The children in the formal operational stage predicted both distributions correctly and were able to justify their predictions. They recognized that the distributions tended to become smoother and more symmetrical as the number of trials increased. In dealing with the uniform distribution these subjects showed an understanding of proportional reasoning, reasoning that is typical of formal operational thought and that was not observed among children of earlier stages.

Another study dealt with the concept of a probability experiment with equally likely outcomes; the material used was a spinner with equal-size sectors, and, later in the questioning, a series of matchboxes of graded weights one of which contained a magnet so that the spinner could be made to land on a particular color. Children were questioned about the outcomes of spinning the spinner both when it was a fair spinner and when the matchboxes were placed on the sectors to influence the outcome.

Subjects in the preoperational stage seemed to believe that it is possible to predict the outcome of spinning the unbiased spinner, even in the face of repeated failure to predict correctly. Sometimes these subjects guessed that the pointer would have a tendency to come back to colors it had previously landed on; sometimes they guessed that it would stop at colors it had not yet touched. Other subjects predicted that the outcomes would follow the order of colors on the card or be related to similarities between colors. Common elements in these and other predictions made by the children were the beliefs that it is possible, at least in principle, to predict the outcomes, and that explanations for the spinner's behavior can be found. Most of the subjects were not surprised by the action of the spinner when it was controlled by the magnet, and many of them explained the new behavior in the same terms as they used in accounting for the outcomes of the unbiased experiment. Although many preoperational children concluded that it was the boxes that made the spinner stop on the color where the box was resting, none of them seemed to recognize this behavior as fundamentally different from the case where the spinner stopped at random. Apparently this is because these subjects believed there to be hidden regularities and reasons for the irregular outcomes of the previous case, too. The phenomenistic and egocentric characteristics of preoperational thought can be seen in these predictions.

When asked to predict the outcome of spinning the fair spinner, the concrete operational children were reluctant to do so, maintaining instead that one can guess but never know for sure where the pointer will stop. This indicates that the concrete operational child has developed a notion of chance (the outcomes are fortuitous and there are no hidden regularities) to replace the preoperational child's concept of caprice (the outcomes are governed by hidden and arbitrary laws). That the concrete operational child is beginning to develop a concept of the distribution as a whole can be seen, for example, in predictions that repeated landing on a single color would be unlikely, and that the spinner landing on one of the several colors not yet touched would be more likely than landing again on one of the one or two previously touched colors. This understanding of chance and these predictions seem to be based on the mental operation of compensation as it relates to the random mixture of possible outcomes. Still, there are limits to these children's ability to quantify their predictions and extend them beyond small numbers of outcomes. For example, several children predicted that with a large number of trials the distributions would be less regular than with fewer trials. This may be the result of focussing on absolute differences in the numbers of times each outcome would occur (lack of understanding of proportionality, which is normal among concrete operational subjects) or of failing to apply the operation of compensation when the outcomes are no longer present or readily imagined (concrete).

When using the apparatus with the matchboxes, all of the concrete operational subjects considered that the systematic stopping of the bar at a magnetized box was caused by some factor which was not present in the earlier trials. Their concept of chance was such that they rejected the idea that the repeated stopping of the spinner in one place could be the result of chance alone. For example, one subject commented that stopping in the same place up to ten times in a row would be normal, but more than ten times would not be normal. In attempting to explain the behavior of the spinner with the matchboxes present the children conducted a number of experiments, and in doing so they ruled out factors such as the speed of the spin, the color of the sector, and the weight of the box. In this experimentation the subjects seemed to be using the mental operation of disjunction in testing whether or not a factor was effective in influencing the outcome.

Formal operational children made responses similar to those of the concrete operational children when asked to predict the outcomes of single trials with the unbiased spinner. When asked about the outcomes of a large number of trials, these children believed that the distribution would become more even and regular as the number of trials increased, reasoning that there would be more opportunity for compensation, a better mixing of the outcomes, or more chances for all of the outcomes to occur. Implicit in their reasoning was an understanding of proportionality, in that while there would likely be differences in the numbers of the different outcomes, these differences would be relatively less important as the numbers increased. In working with the spinner and the matchboxes formal operational children quickly realized that the boxes were affecting the outcomes, and many of them hypothesized that the effect was due to magnetism. Their experimental activities in exploring this situation were qualitatively superior to those of the concrete operational children and included clear dissociation of factors, organized counterproofs, and systematic variation of quantitative factors such as distance and weight. In their experiments and their comments on them the subjects used hypothetical-deductive operations typical of formal operational thought.

Piaget and Inhelder's analysis of the development of concepts of combinatorics begins with the observation that there is an essential logical connection between concepts of randomness and probability and concepts of combinatorics. They argue, for example, that the idea of mixture (as in their study of randomness) is logically equivalent to the construction in a concrete setting of permutations, and that the prediction of the distribution of outcomes of a large number of trials of an experiment (as in their studies using a spinner) logically entails the construction of the combinations (selections with replacement) of m possible outcomes taken n trials at a time. Given this logical analysis, Piaget and Inhelder move on to empirical investigations with the goal of determining whether children's development of combinatorial operations parallels their development of concepts of probability as is predicted by the logical analysis. In general, the empirical studies do support the prediction, and the analysis shows how children's abilities with combinatorial operations are related to the development in three stages of cognitive operations in general.

Included in the empirical investigations of children's combinatorial abilities were tasks involving combinations (unordered selections without replacement) of n items taken two at a time, permutations (ordered selections without replacement) of n items, and "arrangements" (ordered selections with replacement) of three items taken two at a time. The results were quite similar across tasks; in the following section only the first type of task is described in detail.

When the combinations task was presented to preoperational children, piles of red counters (R), white counters (W), and blue counters (B) were used. The children were told that the counters represented little boys, and they were asked to show how the boys could leave two at a time in all possible combinations. At this stage pairs of the same color were permitted, and, in fact, many subjects began with the pairs RR, WW, and BB. Some subjects required prompting in order to go on to pairings of two colors; most subjects required prompting in order to discover the pair made up of non-adjacent colors. Piaget and Inhelder inferred that these children were using seriation in constructing in order the pairs RR, WW, and BB, but noted that beyond this point they seemed to be groping for pairs without a system. They had little success determining whether they had found all possible pairs and considerable difficulty detecting pairs that were repeated.

The concrete operational children were given four different colors of counters (later six different colors) and were asked to make pairs. Then they were asked whether they could find a quicker and more accurate way of doing the task without forgetting or repeating any of the pairs. These children tended to construct pairs first by taking adjacent, non-overlapping pairs (e. g. with colors A, B, C, and D forming AB and CD, and similarly with five or six colors), then by taking adjacent, overlapping pairs (e. g. with A, B, C, and D forming

AB, BC, CD, and similarly for larger sets). Several other systematic methods were used, but rarely were they followed to a complete enumeration of all possible pairs. When concrete operational children did succeed in producing all possible pairs, they most often discovered the final pair or pairs by trial and error. The mental operations characterizing these methods include seriation, juxtaposition, and symmetry, but not multiplication of classes (cross product of sets).

The formal operational children were given the same materials and tasks as the concrete operational children. Their solutions were generally complete and correct, and, with minimal prompting, systematic. A typical solution to the six color problem was a row of five counters of the first color each matched with a counter of one of the remaining five colors, followed by a row of four counters of the second color each matched with a counter of the remaining four colors, and so on. Many of the formal operational children made remarks indicative of their systematic thinking, for example, that there would be six rows of six pairs if pairs of the same color and reversed pairs were allowed, and that there was one less pair in each successive row. The main differences between their performances and those of the concrete operational children were that the solutions were completely systematic and that the operation multiplication of classes (cross product of sets) was used. This operation may be thought of as the formal (abstract, hypothetical) construction of a series of series.

In summary, the findings of Piaget and Inhelder (1976) indicate that children's development of the related concepts of randomness, chance, and combinatorial operations occurs in stages which can be characterized by the mental operations that children in each stage typically use. Although preoperational children have certain intuitions and anticipations, they have not developed the logical operations which render some events certain (i. e. deriving by logical operations from necessity) and, by contrast, other events fortuitous (i. e. resulting from a random process). With reference to specific tasks it was shown that preoperational children often suspect reasons and regularities where there can be none and fail to suspect an external cause when bias is present. Their reasoning is prelogical. Concrete operational children have begun to develop a notion of chance, a domain that is unpredictable and antithetical to the domain of logical operations and the predictability that results from them. They maintain the unpredictability of individual random events and use compensation and symmetry as a basis for their limited understanding of the distributions of outcomes of repeated trials, but their lack of understanding of proportionality prevents them from grasping the regularities and evenness in distributions for very large numbers of trials. In combinatorics their methods of solution are somewhat systematic, based on concrete operations, but not completely systematic. Formal operational children have begun a synthesis of chance and operations in their recognition that while individual outcomes of a random process are unpredictable, distributions of the outcomes of repeated trials are predictable. In this process formal operational thought which ties together systems of concrete operations, hypothetical-deductive reasoning, proportionality, and a completely systematic approach to problems of combinatorics are the key elements.

The Work of Fischbein

A book titled The Intuitive Sources of Probabilistic Thinking in Children by E. Fischbein was published in 1975. Like the volume by Piaget and Inhelder (1951, 1976) it deals in great detail with the development of probability ideas in children of preschool, elementary school, and early secondary school age. Its theoretical basis owes a great deal to the Piagetian analysis, but it is at the same time both broader, in the sense that points of view other than the Piagetian are considered, and more focussed, in that it deals primarily with the nature and role of intuitions.

Fischbein describes intuitions as "cognitive acquisitions which intervene directly in practical or mental actions, by virtue of their characteristic immediacy, globality, extrapolative capacity, structurality, and self-evidentness," (Fischbein, 1975, p. 117) and as "forms of immediate cognition in which the justifying elements, if any, are implicit." (Fischbein, 1975, p. 5) Intuitions may be classified as either primary (i. e. derived from the experience of the individual without instruction) or secondary (i. e. formed by education, usually in school) and as either affirmatory (i. e. embodying knowledge of the world that is accepted as self-evident) or anticipatory (i. e. giving a global view of a problem situation which precedes the detailed solution process). With respect to cognitive operations intutions may be antecedent, facilitating the development of operations and the formation of concepts, or concurrent, facilitating the continuity and fluency of operations, or post-operational, serving to synthesize global cognitive organization and facilitating transfer.

Fischbein's presentation of information about children's development with respect to probability includes consideration of both their overt behavior and their intuitive and operational thinking. In the analysis Piagetian terminology and organization are used extensively, and the result is, on the whole, an enhancement rather than a refutation of the Piagetian depiction.

Fischbein assesses the preoperational child's understanding of probability as follows:

> The probabilistic judgement of pre-school children is precarious because they do not yet have an adequate conceptual framework. Their responses can only express intuitive, pre-operational estimates. The play of influences at the cognitive level can easily mislead the child in making decisions, because of the absence of any relevant conceptual control.
>
> (Fischbein, 1975, p. 120)

This is quite in accord with the views of Piaget and Inhelder (1976). But Fischbein then elaborates on the nature of these intuitions, first citing results from a number of studies in which children were asked to observe one trial of a probability experiment then predict the next outcome, then observe, then predict, and so on repeatedly. Many forms of this experiment have been carried out with children ranging in age from preschool to adolescence, using various frequency distributions, both with and without reinforcement. Two particular kinds of behavior have been observed: probability matching, where the subject's predictions are random but include outcomes that have been observed in proportions that match the respective relative frequencies, and overshooting, where the subject constantly predicts the single outcome which has had the greatest frequency. It is a well documented fact that statistically significant probability matching behavior occurs even in preoperational children, and Fischbein attributes this behavior to primary intuition. Fischbein also cites studies in which preschool children responded correctly to questions concerning the evaluation of chances. He claims that in these studies the questions were specifically and unambiguously posed in these terms and that the responses show that the child has correctly grasped the problem and is truly expressing an intuitive probabilistic judgement.

Since the extent and precision of probability matching behavior increase during the period when children are in the stage of concrete operations, Fischbein concludes that intuitions of relative frequency are developing, even though proportional reasoning is not evident until the later formal operational stage. Another arena in which the effects of intuition can be seen in concrete operational children is in their solutions of combinatoric problems. The trial and error procedures children use are admittedly not completely systematic, but intuition appears to be operating in the children's selection of the operations (e. g. seriation, symmetry) they will use. Furthermore, the fact that children can with the help of instruction assimilate certain useful enumerative procedures and do exhibit transfer of them suggests that antecedent intuitions of combinatorial operations are present.

Reflection on the nature and role of intuitions is also important in considering the formal operational child's concepts and abilities in probability. Fischbein asserts that "the synthesis between the random and the deducible is not realized spontaneously and completely at the level of formal operations," and that "the operational structure of formal thought alone cannot make chance intelligible, even though it can provide the schemas necessary for this, namely combinatory capacity, proportionality, and implication." (Fischbein, 1975, p. 127) In the list of necessary conditions for a complete development of concepts of probability, Fischbein would include intuitions as well as instruction which capitalizes on intuitions and which anticipates aspects of probability that students may find counter-intuitive.

Other Investigations

A number of other researchers have explored children's probability concepts in the absence of instruction, as well. Many of these studies have employed quantitative analyses and have yielded information about relationships between measures of probability concept development or application and variables such as age, grade, IQ, and achievement. Reviews of selected studies in this category are presented in this section.

Jones (1974) explored the status of the following five concepts of probability: outcomes in a sample space, most likely event in a sample space, most favorable sample space for a given event (among sample spaces with equal total numbers of outcomes), sample spaces equally favorable to a given event, and most favorable sample space for a given event (among sample spaces with equal numbers of favorable outcomes). Embodiments of these concepts included two kinds of spinners and containers of discrete objects. Subjects were students in grades one, two and three classified according to sex, grade, and IQ level. Univariate and multivariate analyses of variance were used to detect differences due to sex, grade, IQ, and embodiments. Jones observed that most children in grades one, two and three have begun to acquire some knowledge of probability, and the extent of their knowledge generally depends upon their grade and IQ, but not their sex. He also noted that these children responded differently to situations that were mathematically isomorphic but used different embodiments. From this he concluded that it would be appropriate to introduce some topics in probability into the primary mathematics curriculum, and that in doing so a variety of embodiments should be used.

Doherty (1965) studied the status of four probability concepts (viz. the concept of sample space, the probability of a simple event, the probability of the union of disjoint simple events, and the concepts of mutually independent and mutually exclusive events) in fourth, fifth, and sixth grade students who had not received any formal instruction in probability. Paper and pencil tests of these concepts were constructed, and data were also collected on the age, sex, grade, mental age, arithmetic achievement, overall achievement, and teacher rating of each child. Analysis of variance was used and tests of the significance of main effects were reported; interaction effects were not reported. The overall means for the four concepts ranged from a low of 31% (probability of union of events) to a high of 60% (probability of a simple event), but this difference was not found to be significant. Differences due to age and sex were also not significant; the differences due to grade, mental age, arithmetic achievement, overall achievement, and teacher rating were found to be statistically significant. In each of these cases probability test performance improved as the level of the independent factor increased. Doherty concluded that these upper elementary school children had acquired considerable familiarity and ability to deal with these probability concepts. It should be noted, however, that qualitative assessment of the subjects' understanding was not included in the study.

A similar investigation by Leffin (1968) dealt with three concepts of probability (viz. sample space, probability of a simple event, and probability quantification) in students in grades four to seven before formal instruction in probability. Three factor multivariate analysis of covariance was used, the dependent vector consisting of scores on investigator-constructed tests of the three concepts, the covariates being standardized achievement test subtest scores, and the independent factors being IQ, sex, and grade. The main effect for each of the independent factors was significant, but none of the interactions was significant. Echoing Doherty, Lefflin concluded that the students had acquired considerable knowledge of the concepts under investigation and ability to apply them in a variety of situations.

A third study along these same lines was conducted by Mullenex (1968). A 28 item group test of probability concept understanding containing questions on randomness, probability (as opposed to certainty), quantification of probability, and probability distributions was constructed and administered to students in grades three to six. Independent variables included age, sex, IQ, and standardized achievement test subscale scores. Using multiple linear regression techniques it was found that age, general achievement, vocabulary skill, work-study skill, and age interactions were not significant predictors of total scores on the probability test, but sex, reading achievement, and problem solving achievement, while also not significant, did approach significance. Large positive correlations between some of the independent variables may have eroded the predictive power of individual variables, since each variable was tested in the presence of others. Mullenex noted the students' enthusiasm for work with probability concepts and concluded that probability concepts were suitable for inclusion in the elementary school mathematics curriculum.

Kelsey (1980) examined adolescents' development of notions of chance, probability, and combinatorial operations using interviews where Piagetian-type task protocols were presented to students in grades seven, nine, and eleven. Qualitative judgements were involved in the scoring, but results of each of the six tasks were apparently treated as dichotomous variables. Scalogram analysis and chi square analyses were used to determine the order of difficulty of the tasks. The probability quantification task was most difficult, but the analyses did not support a clear ordering of all the remaining tasks. Kelsey judged these students' understanding of probability and combinatorial concepts to be poor, but did not offer any explanation for the difference between his negative assessment and the generally positive evaluations of other researchers such as Doherty, Leffin, and Mullenex.

Instruction in Probability

In recent years a great deal has been written about the teaching of probability; much of it has dealt with introducing probability to chilren of elementary school age. Some of this literature can be classified as advice and opinion offered by persons wanting to share their reflections on their experiences. Many of the articles in the National Council of Teachers of Mathematics yearbook, Teaching Statistics and Probability, (Shulte, 1981) and many of the 142 citations included in its bibliography fall into this category. Other studies have dealt with the questions of how probability can be taught and of how it can be taught most effectively in a more thorough and systematic way. Certain reports in this second category are reviewed in this section.

Ojemann and his colleagues (1965) studied the effects of teaching probability to third grade children using a series of five 30 minute activities on five consecutive days. The research design involved pretests and posttests of the treatment group of an untreated control group. In devising the learning program they considered probability from the point of view of prediction in general, and they used a variety of situations in which the prediction of uncertain events is involved. In planning the instruction for third grade children they restricted themselves to the following three types of situations: (1) ones in which the number of factors influencing the outcome is small and information about them is available in simple form, (2) situations in which a number of significant factors have to be considered and their influence estimated, and (3) situations in which little or no information about significant factors is available, but there are opportunities to make tentative predictions while gathering information. The learning program consisted of experiences designed to promote the following abstractions: (1) that when a random selection is made from a group of equally available alternatives, each alternative has an equal chance of being chosen, but (2) when there are more of one item than of another, the former has the greater chance of being selected, (3) that more information about how a random experiment behaves is useful and that one needs to know what factors are involved and, if more than one factor is involved, how their effects are combined, and (4) that as one gains experience with a random process one can gather information about it that will be useful in predicting later outcomes. Although the study was concerned with the development of students' understanding of concepts, the posttests used involved only a restricted range of behavioral outcomes. In the first 25-question posttest, situations were described involving objects identical except for their color hidden in a bag. Students were told how many objects of each of two colors there were, and they were asked "If I reach into the bag and pull out one of the objects, what color do you think I would get?" Each item was scored correct if the child predicted the color of which there were more objects or responded "I don't know" or "You can't tell" when the numbers of each color were equal. Surprisingly, the experimenters did not elicit or score responses of tho sort, "You can't tell for certain, but color X is more likely than color Y." The second posttest of 24 items was similar, but it involved predicting the outcome of drawing a card from a deck containing given numbers of cards each marked with either a square or a circle. A third posttest was geometrical in nature and involved prediction of a shape as the parts of the shape were revealed. The final posttest involved repeatedly predicting the outcome of drawing a washer out of a box (with replacement). The children were told that the box contained washers of two specified colors but were not told how many of each color there were. This test was seen as relevant to children's gathering and use of information as they gained experience with a random process.

The comparison of results on these tests by subjects in the experimental group and the control group indicated significant differences in favor of the experimental group in ability to predict, and a non-significant tendency on the part of experimental group students to gather then apply information more effectively in the fourth test. Although this confirms in a general way that the tasks and discussions were suitable and effective, it is unfortunate that no qualitative data are reported and that the outcomes measured are so narrow and so different from measures of attainment of the concepts that the program set out to introduce. Despite a promising start with a clear view of the "abstractions" (concepts, generalizations) that it was intended would be developed in children, the question of whether these concepts actually were developed in the subjects remains unanswered.

McLeod (1971) investigated the feasibility of teaching probability to second and fourth graders. His experimental lessons used the drawing of colored balls from a bag and dealt with the workings of the law of large numbers, the prediction of outcomes of a probability experiment, the principle of independence of events, and the application of probabilistic vocabulary to everyday expereinces. Symbolization and calculation of probabilities were avoided. Some students in the study were taught by laboratory participation and others by teacher demonstration. To control for maturation and the effects of taking the tests, one group was given the pretests and the posttests but no instruction; another group was posttested only.

McLeod observed that most of the second graders and most of the fourth graders were able to apply the concepts likely, more likely, equal chances, less likely, and unlikely even before instruction began. Both the groups taught by laboratory participation and by teacher demonstration significantly outperformed the untreated control group on the posttest, but no significant differences were found between the treatment groups and the group that was only pretested and posttested but not taught.

Another exploration of the teaching of probability concepts to elementary school students was undertaken by Gipson (1972). Concepts included in this study were the possible outcomes of an event (experiment?), listing the number of possible outcomes for a finite sample space, the probability of a simple event, the probability of an impossible event, equally likely events, and probability estimates. The experimental instruction was developed through a series of stages and was taught in its final form on an individual basis to eight children in grades three and six. Some of the teaching sessions were videotaped and the others were audiotaped. From this experience Gipson concluded that it was feasible to teach these probability concepts to children in these elementary grades.

Games and Computer Gaming

An extensive review of research literature related to the cognitive effects of games on mathematics learning was carried out in the late 1970's by Bright, Harvey, and Wheeler (1979). In the review they considered three kinds of uses for games: the pre-instructional use (where games are used to introduce a concept or skill, or as an advance organizer, or as a preliminary experience of some other sort), the co-instructional use (where games are used alone or in conjunction with other learning activities after introduction of a concept or skill but before its mastery), and the post-instructional use (where games are used to promote the retention of previously learned material). It is interesting to note that in none of the 55 studies included in the review was instruction in probability the educational purpose of the game or the focus of the study; nonetheless, probability ideas could be profitably applied in many of the game situations.

In order to remedy this defect, the use of games and children's incidental learning of the probability concept fairness were included in a later study by Bright, Harvey, and Wheeler (1980, 1981). This study involved seventh grade students who played games during 20-minute sessions held twice weekly for four weeks. Although no explicit instruction in probability was given by the teachers (since this aspect of the study was conceived of as an exploration of the effects of games used "pre-instructionally"), there was a statistically significant ($p < 0.01$) increase from pretest to posttest in scores on a test in which students were to identify whether one or the other or neither of two game situations gives the better chance of winning. The investigators claim that "whatever the students learned was from playing the games. This suggests that some games can be used effectively before formal instruction." (Bright, Harvey & Wheeler, 1981, p. 59)

Although it is tempting to accept these conclusions because they seem reasonable and warranted on the basis of the statistical data provided, a closer look at the more detailed report (Bright, Harvey & Wheeler, 1980) reveals some facts that give quite a different picture. First, the part of the treatments that involved probability concepts was confined to only a part of one of the eight game-playing sessions, i.e. it took less than 30 minutes. Second, the pretest and posttest were identical, and the average score increased by less than two items on the 28 item test. Since there was no untreated control group that took this test twice, there is no way of refuting the quite reasonable alternative hypothesis that this small increase resulted from practice or maturation effects. Third, even on the posttest students gave correct answers to only about 63% of the items. A treatment of such short duration could hardly have been expected to have an educationally significant impact, and the outcome that was observed hardly deserves to be called a significant amount of learning resulting from an effective treatment.

The constructivist view of the importance of action to learning does not imply that physical experience alone produces cognitive change. Components such as questions and discussions can, by serving to draw students' attention to relevant aspects of the experience, produce effects that might not have resulted from experience alone. Although this report would seem to be relevant to the present investigation, close examination reveals that it lacks power and that its statistically significant result is probably not educationally significant. The fact that the games were used as "pre-instructional" experiences only without discussion, rather than "co-instructionally" diminishes their relevance to the present investigation as does the fact that the study is quantitative and product focused rather than qualitative and concerned with educational processes.

Two dissertation studies by students at Temple University investigated the effects of the use strategy games on children's problem solving ability as measured by the Krulik Problem Solving Achievement Test (KPSAT). Both studies are based on the view that the processes involved in developing a winning strategy for playing a game and the processes involved in solving a non-routine problem are similar, and that experiences with the former could therefore be expected to transfer to the latter. The first study, by Goldberg (1980), dealt with seventh grade students, and the second study, by Fluck (1982), was done with fifth grade students who played computational/strategy games, that is games in which computational practice, chance, and the ability to develop a strategy for winning are all involved. In both studies students who played strategy games had significantly higher posttest scores on the KPSAT than control group students. In Fluck's study no instruction in probability was explicitly included and no measures of children's understanding or application of probability concepts were used, but certain probability notions are implicit in the games and in the development of strategies for playing them. In describing the experimental treatment, Fluck laid great emphasis on the role of teachers' discussion of strategies with students. The teachers were trained to help children to develop their own strategies and encouraged not to tell students a winning strategy. In this training Fluck stressed the need to ask questions and otherwise draw children's attention to relevant features of the games. Although the outcomes focussed on are different from the concerns of the present investigation, the teaching methods used have many similarities to the activities used in this study. The fact that these methods have been shown to have had a significant positive impact on children's cognitive (and perhaps meta-cognitive) abilities is encouraging and lends support to testing them in a different context.

In adding some qualitative reflections to his quantitative study of first, second, and third grade students' probability concepts, Jones (1974) noted that some children ascribed their winning or losing of games not to chance but to personal attributes of their opponents. Comments such as "Of course, you let me win" and "Big people always win games" were interpreted as evidence that some children may not think about games in terms of probability and chance because their experiences suggest to them that other factors controlled by older or more knowledgeable players matter more. Since in the present study a computer both controls the roll of the dice and determines the moves of the opponent, there is reason to wonder whether a similar line of reasoning might lead children to believe that their experiences playing Capture are similarly non-random. Since many people (both adults and children) conceptualize computers as powerful "brains," they might view their situation as competition against an opponent that has knowledge they cannot hope to have.

A study by Brod (1972) investigated junior high school students' attitudes toward their teachers and toward the computer when they were involved in a computer assisted instruction (CAI) program of remedial mathematics drill and practice. Brod observed that where the CAI system was given normative support by school officials authorizing its exercise of power to monitor students' task specific behavior and where teachers did not have control over the system, the children were likely to form authority relationships comparable in nature and degree to those which they normally formed with their teachers. Brod described these relationships as undermining the teachers' authority and he noted that they tended to be more problematic with students in their first year than with students in their second or third year of

CAI experience. Since the present study does not involve relinquishing human control over instructional decision making, it is not likely that students in this study will develop a similar relationship with the computer as an authority figure. Nonetheless, Brod's study does suggest that students do respond to the apparent personality of the computer and do regard it as having a certain competence.

These two studies suggest that children's attitudes toward the computer and their view of its knowledge and authority may need to be explored in the present study. If students should believe that the game situation is "rigged" or is controlled by knowledge not available to them (e. g. knowledge of future rolls of the dice) then they might not think of the situation or act on it using probability concepts as they would if they were playing a game with a human opponent.

Peelle (1971) studied the learning of fifth and sixth grade students as they played variants of Nim on an interactive computer system. The children played against a computer simulated opponent, and they were able to select the opponent's level of game playing ability, to choose whether the computer simulated player was "learning" or "static" (i. e. exhibiting progressive adoption of an optimal strategy, resembling some human learners, or playing consistently according to a single strategy), and to direct the computer to "play harder" or "play easier." Although no statistically significant differences in variables constructed to measure learning were found, comprehensive data were gathered from the game playing sessions, differences in the effectiveness of the programs were observed, and hypotheses for further investigation were identified.

Kraus (1980) studied the use of problem solving heuristics by novice and experienced problem solvers as they played variants of Nim against a computer simulated player. The computer simulated player was designed to play the games adjusting its level of play on the basis of the human opponent's win/loss record. The subjects were audiotaped as they thought aloud while playing the games. Of the novice players, who were eighth grade students, about half did not regard the game playing situations as problems to solve, and their heuristics were almost exclusively random trial and error. Other novice problem solvers did regard the games as problems; they made limited use of working forward, systematic trial and error, searching for patterns, reviewing previous work, and using subgoals, and made extensive use of random trial and error. In a later phase of the study a second computer program was developed to simulate the human subjects' play. Ten parameters were used to adjust this program's play so that it would correspond to the individual subjects' actual performance. It was found that the scores of the second computer simulated player were lower than but significantly correlated with the scores of the corresponding human players, and it was concluded that the use of heuristics accounted for a significant part of the differences in problem solving performance.

The studies of Peele and Kraus are not directly applicable to the present investigation, since they deal with different games (variants of Nim), and they are not concerned with the the development or application of probability concepts, but they do have striking methodological similarities to this study. They show that it is feasible for children to play games against a computer simulated opponent, they suggest that children's play may be characterized by the extent to which they use certain strategies identified in advance, and they suggest that it may be useful to adjust the computer simulated player's level of play to suit the child's level of play. Kraus's use of the thinking aloud technique seems to have been fruitful in making it possible to identify the heuristics that the problem solvers were actually using. In both studies the children were involved in solving the problem of how to win a strategy game, yet many of the eighth grade students in Kraus's study did not recognize the situation as a problem solving situation and did not apply to it problem solving heuristics other than random trial and error. This suggests that interviewer questions about strategies for games might be necessary and useful in cuing the children to search for strategies.

Summary

The research reviewed in this chapter holds several implications for the present investigation. The research on children's development of understanding of probability shows that it is reasonable to expect that the children to be studied, who are in grades four, five, and six and who range in age from about 10 to 12 years, will have certain understandings and abilities related to probability that they can apply in playing and developing strategies for Capture. These understandings and abilities are related to the development of intuitions and cognitive operations, and this development has been going on for some time, even in the absence of formal instruction instruction in probability. In particular, these children can be expected to be at a stage where they recognize the unpredictability of events such as the rolling of dice and have an appreciation of the fact that even among uncertain events some outcomes may be more likely than others. Although these students may not be expected to apply spontaneously or consistently their developing and not completely systematized abilities to enumerate all the outcomes of a probability experiment with compound events, they may be expected to have some degree of skill and an ability to benefit from intstruction in this area.

The research on computer gaming suggests that having the children think aloud as they play against a computer simulated opponent and measuring the extent to which their play is consistent with known strategies for playing are two promising techniques with which to investigate their thinking.

CHAPTER 3

METHODOLOGY

In this chapter the methodology of the study is presented in several stages. First are general considerations including comments about the relationship between the questions under investigation and the methods which are valid, appropriate and useful for investigating those questions, and a perspective on the recent history of research methods in the field of mathematics education. This leads to a description of several research methodologies collectively referred to as Soviet-style natural experiments and an explanation of and rationale for the particular methodology used in this study. Next, the procedures of the study are outlined, and the means of gathering and analyzing the data, both quantitative and qualitative, are described. Finally, the students who took part in the study and the methods that were used to select them are described.

General Considerations

It is essential that there be harmony between the questions a research study sets out to answer and the methods that are chosen to find those answers. In the recent history of mathematics education research there has been a noticeable evolution involving changes in the types of questions considered along with changes in the types of methodologies that have been employed. In describing these changes

Davis and McKnight (1979) observe that over the past 20 years or so researchers have come to reject the views that one cannot know what is going on inside people's heads and hence that one must deal only with observable behaviors and that one should interpret observable behaviors in terms of numeric-valued variables such as indices of intelligence, achievement, or attitude. Two reasons given for rejecting these views are, first, that they do not reflect what mathematics educators really care about, since mathematics is a matter of ideas developed in the mind -- ideas that may be verbalized, written down, or otherwise reflected in behavior to a greater or lesser extent -- and, secondly, that they do not produce research reports with enough specificity and descriptiveness to be useful in suggesting practical courses of action. Research concerned with the identification and analysis of flaws or "bugs" in students' procedures and research aimed at the interpretation of students' behavior in terms of metaphoric models of human information processing are cited as examples where the questions and methods are quite different from questions which are answered by statistical studies of measures of more narrowly defined educational outputs.

In another paper dealing with his opinions about what is needed in North American research in mathematics education, Davis (1979) has identified three alternative kinds of research. The first is the sort of "abstract/statistical" experimental research that has predominated until recently. In this sort of research variables (measurements of behavior which reflect constructs such as intelligence, achievement, or attitude) are defined, and the relationships among variables or the influence of treatments on variables is the focus. An essential feature of this sort of research is that it is concerned far more with products than with processes.

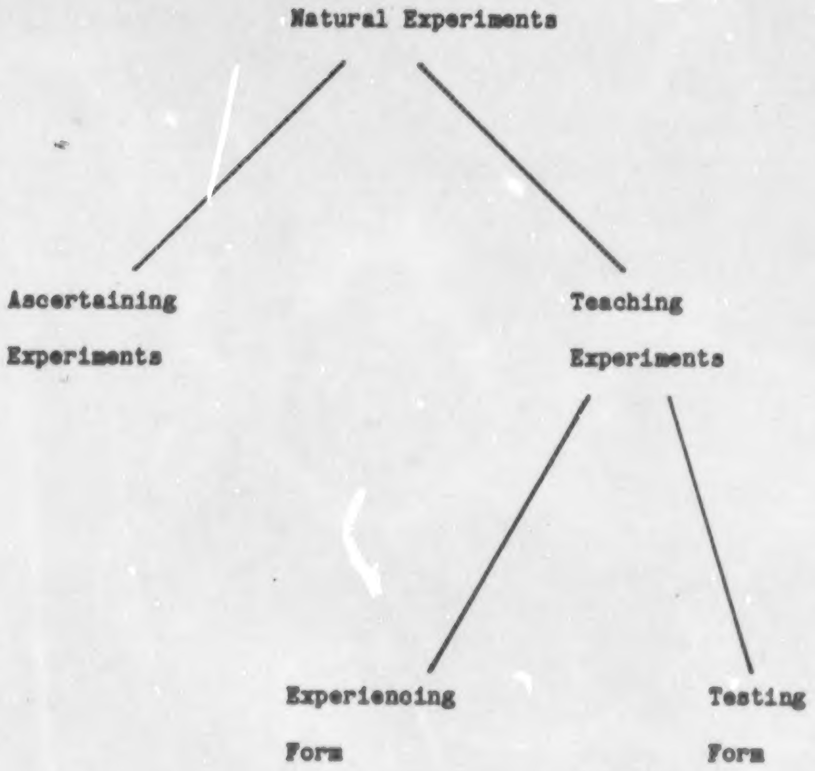
A second and fundamentally different alternative is the approach used by Piaget. In this sort of research the observation and detailed description of behavior with respect to special tasks leads to hypotheses about the nature of and development of children's knowledge and intelligence. In assessing the impact of this alternative on mathematics education Davis notes two important drawbacks: first, that while the Piagetian tasks illuminate aspects of children's thinking and understanding in general, they often involve tasks that are quite remote from school-type tasks such as solving word problems or doing numerical computations with algorithms, and second, that (through no fault of Piaget's) Piagetian studies have tended to be interpreted negatively (as describing cognitive tasks that children at various levels cannot do) rather than positively (as showing the richness of the knowledge and cognitive abilities that children develop even without having been taught). Furthermore, Piagetian research usually does not attend to what children can learn or what changes in cognitive structures occur as the result of specifically planned experiences. As Krutetskii has observed, "Not only what a child has already learned but also what he is capable of learning is essential." (Krutetskii, 1976, p. 96)

The third alternative that Davis identifies is the Soviet style of research that uses school-related tasks in task-based interviews. It is this sort of research methodology that has been employed in the present investigation. The following section explains various forms of the Soviet methodology and presents a rationale for the particular research methodology that has been chosen for this study.

The research methods which North Americans label as 'Soviet' have been used in the Soviet Union for more than two decades, but they are only belatedly becoming widely known on this continent. The writings of Kalmykova (1967) and of Krutetskii (1976) and a fourteen volume collection of articles by various Soviet authors, Soviet Studies in the Psychology of Learning and Teaching Mathematics, are important primary sources in English translation. North Americans who have described, analyzed, and commented on Soviet research methods in mathematics education include Kantowski (1978, 1979) and Rachlin (1979).

The Soviet research methods considered here are all referred to as 'natural experiments.' Natural experiments are usually either 'ascertaining experiments' or 'teaching experiments.' Teaching experiments are further categorized as being either of the 'experiencing form' or the 'testing form.' This classification scheme, pictured in Figure 3.1, uses the terminology of Kantowski (1979). Kalmykova (1967) classifies natural experiments in the same way but uses the terms 'testing experiment' or 'searching experiment' in place of the term 'experiencing form of teaching experiment' and the term 'verifying experiment' instead of 'testing form of teaching experiment.' These differences appear to result from difficulties in translation; the terms used by Kantowski seem clearer and more descriptive.

Figure 3.1: Classification of Soviet research methodologies


Natural Experiments
Ascertaining Experiments
Teaching Experiments
Experiencing Form
Testing Form

The natural experiment has been developed to overcome some of the drawbacks of educational research carried out in the laboratory. Instead of having children perform unusual, artificial learning tasks in an unfamiliar environment, the natural experiment is undertaken in the child's own school setting using tasks like school assignments presented as alternative or extra-curricular activities. The experimenter is usually in close contact with the subjects' teachers and often takes pains to become acquainted with students before starting experimental work with them. While Western researchers might view features such as these as potential sources of contamination or bias, the Soviet position is that they increase the studies' relevance and applicability and reduce interference from novelty effects. The experimenter strives for consistency in the ways subjects are treated and tries to control factors other than those under consideration in the experiment. Objective data are collected in the form of samples of children's work and by means of tape recorders and video or movie cameras, but care is usually taken to conceal the apparatus or at least make it inconspicuous.

As the name suggests, the purpose of an ascertaining experiment is to ascertain the status of knowledge, or behavior patterns, or habits of work, or modes of intellectual activity peculiar to individuals under given conditions. Sometimes ascertaining experiments focus on error patterns as a preliminary step before diagnosis. For example, Krutetskii's volume on the nature of mathematical abilities resulted from an extensive series of coordinated ascertaining experiments in which children were observed as they solved experimental problems developed by the experimenter. Krutetskii comments, "Experimental problems ought to fulfill their direct purpose: solving them should help to clarify the structure of abilities. In other words, as the problems are solved, those features of mental activity that are specific to mathematical activity should be manifested." (Krutetskii, 1976, p. 91) Krutetskii and others using ascertaining experiments emphasize the need for careful development and selection of the tasks and of the prompts, hints, and direct questions that may be used in exploring students' knowledge and abilities.

Teaching experiments are often used after ascertaining experiments in the second phase of a program of research and development. Having formed a view of the status of knowledge or cognitive abilities in children, researchers then turn to the question of how teaching can influence their development. Since it is believed that mental abilities are formed in activity (the constructivist view) and manifested in activity, activities are used to explore mental processes and their development. Just as the ascertaining experiment uses tasks to reflect the state of knowledge or cognitive abilities, the teaching experiment uses instructional activities of various kinds to reflect the development of knowledge or abilities. The experiencing mode of teaching experiment is used to explore and refine particular instructional activities and techniques without explicit comparisons to alternatives. In the testing form of teaching experiment evidence is sought that a specific method or principle of instruction is superior to others. In this respect it is similar to

Western studies in which the effectiveness of two or more treatments is compared.

The report of a teaching experiment usually takes the form of a narrative with analysis and interpretation of selected behaviors. The data are mainly qualitative and the analyses subjective; quantitative data, when reported, are used to support qualitative judgements. Descriptive statistics are sometimes included; inferential statistics are seldom used.

Rachlin has characterized the conduct of Soviet research as "a loosely-scripted one-on-one audio-taped interview (in which) the subject is asked to think aloud as he or she solves a large number of curriculum related tasks." (Rachlin, 1979, p. 123) The thinking aloud aspect is seen as crucial in finding out about "what is going on inside people's heads," but it is not without its drawbacks. As Krutetskii (1976) notes, many children are not accustomed to thinking aloud, and they find it unnatural and unsettling; sometimes children take the request to think aloud to mean that they should explain their final solution to the problem rather than externalize their thinking as it proceeds; and sometimes children give detailed reports of their observations about their thinking (often colored by their beliefs concerning what the experimenter expects to hear). Krutetskii then briefly describes some introductory comments and instructions given to students to overcome these difficulties, and he claims that in most cases his subjects were able to think aloud with hardly any disturbance to their natural thought processes.

Ericsson and Simon (1980) used human information processing theory as the basis for an extensive analysis of methodologies in which subjects give verbal reports of their thinking. The major conclusion of the analysis is that verbalizations of information which is in short term memory and which is being attended to (or heeded) are reliable data. Although the request to think aloud may affect a subject's speed in problem solving, it does not change the internal structure of thought processes so long as the following three conditions are met: primacy (i. e. performing the task and verbalizing about it are given equal importance), currency (i. e. information is verbalized at the time that it is attended to), and specificity (i. e. subjects are asked to tell about particular events rather than make generalizations).

It is always important to be aware of the potential for distortion inherent in asking children to think about and talk about their thinking, but more so in an ascertaining experiment than in a teaching experiment. In the present study questions which draw students' attention to their thinking and to features of the situation which they might or might not otherwise notice are considered to be an important part of the experimental instructional method, and the effect of using specific questions and lines of questioning is a main focus of the study.

The seven major questions under investigation in this study (listed in Chapter 1) have both qualitative and quantitative aspects, with the qualitative predominating. A natural experiment designed to answer these questions would need to have certain features of an ascertaining experiment (for example, in describing children's spontaneous use of strategies) and certain features of a teaching experiment (for example, in exploring the effects of adults' questions on children's game playing behavior and their discussion of it). On balance the features which suggest a teaching experiment in the experiencing mode seem most salient.

The Plan of the Study

In this teaching experiment a series of individual interviews was conducted between children in grades four to six and the investigator. The children and experimenter worked in a small room which also contained an audio tape recorder (made inconspicuous but not concealed) and an Apple II Plus microcomputer system comprised of a monitor, CPU/keyboard unit, and two disk drives. In the first session with each child the experimenter explained that he teaches teachers about mathematics and that he also works on creating materials for children to use in school, including computer materials. He defined his purpose as an attempt to find out how children think and how they react when they play two games he had invented, and he asked whether the child would be willing to help him by playing the games and discussing them with him. It was also explained that he wanted to make records of the session, and that an audio tape recorder would be used for this purpose. A positive response from the child was taken as evidence of informed consent from the child. Informed consent of the parents was obtained by means of the letter and form contained in Appendix 1.

The experiment proceeded by having the child use a program that presents the rules of Capture 1, and then having the child play Capture 1 several times. Discussion (involving questions described later in this chapter) was used at various points during play. The child also played and discussed Capture 1 in a second interview session on the following day. In a third session the child learned the rules for Capture 2 then played and discussed it with the experimenter. Each child was interviewed in three sessions on three consecutive school days; each interview lasted about 30 minutes.

Quantitative Data -- Collection and Analysis

In this study quantitative data are collected and stored by means of routines embedded in the game programs, which are written in the Applesoft version of BASIC. The 30 elements of a string variable array are used to store information about each of the 15 moves of each of the two players. Each string is 41 or 42 characters long and is constructed as follows. The first character is '*,' a delimiter used to prevent Applesoft from truncating the string by deleting leading blanks. The next 37 characters correspond to the positions 0 to 36 of the board as it appears while the player is choosing his or her move. These characters are 'A' if Apple holds the position, 'X' if the position is held by the other player, and blank otherwise. The next two characters correspond to the two dice outcomes (i. e. two integers from 1 to 6 in Capture 1, or an integer from 1 to 6 and an operation symbol +, -, or X in Capture 2). The final digit or digits represent the position (0 to 36) that the player has chosen in the current move. At the end of each game the program creates a sequential text file on the diskette containing the names of the two players, the date on which the file was created, and the 30 element data array. Each such file is assigned a unique filename, and another sequential file called DATA INDEX is updated to show the number of data files currently on the diskette together with the dates on which they were created and the names of the players involved.

This data management system makes it possible to retrieve from the diskette a complete record of each turn of the game including the name of the player, the board position during the move, the roll of the dice, and the choice that was played. The purpose for collecting these data is to be able to compare players' statements about their strategies or hypotheses about players' strategies with players' actual overt behaviors. In particular, analyses have been carried out of the extent to which children's play is consistent with four specific strategies identified in the following section.

There are four strategies for playing Capture 1 and Capture 2 which are known to increase a player's probability of winning. Corresponding to them are integer valued variables S1, S2, S3, and S4 which take the value +1 when a player's move is consistent with use of the strategy, -1 when a player's move is inconsistent with use of the strategy, and 0 in cases where the strategy does not apply or cannot be used.

Strategies 1 and 2 are based on the rule that allows a player to recapture positions previously held (Rule 5 in Capture 1, Rule 6 in Capture 2). There is no advantage to recapturing a place a player already holds, and doing so uses a turn that could be used to capture another position. This is the reason for Strategy 1, avoid self-recapture. Strategy 1 and the associated variable S1 are defined in Table 3.1. Capturing a position currently held by the opponent is advantageous because it increases the player's score by one and reduces the opponent's score by one for a net gain of two. Strategy 2, attempt opponent-recapture, and the associated variable S2 are defined in Table 3.2.

Strategy 3 is based on the fact that it is possible to capture some board positions with several different dice roll outcomes, while other positions can be captured with only a small number of dice rolls. There are a few board positions that cannot be captured no matter what. Figure 3.2 contains two graphs showing the number of different dice outcomes which permit the capture of each of the board positions in Capture 1 and Capture 2 respectively. An important implication of this fact is that the probability of an opponent being able to recapture a position a player holds depends upon the number of ways in which the opponent might effect the recapture (i. e. the number of different dice roll outcomes that would permit it) and the number of turns remaining in the game. Tables 3.3 and 3.4 give these probabilities for the two games respectively. An examination of these two tables reveals that in the early stages of a game the probability that the opponent will be able to recapture a given place is quite high; with 12 or more turns remaining these probabilities range from about 0.5 to virtually 1, depending upon the specific position. Toward the end of a game these probabilities decrease and have a wider range; for example with two turns to go the probabilities range from

Figure 3.2: Number of dice roll outcomes which permit the capture of each board position in Capture 1 and Capture 2

```
  Capture 1                     Capture 2


 0 ******                      0 ************
 1 ***********                 1 ********
 2 ***********                 2 **********
 3 **********                  3 ***********
 4 *********                   4 *************
 5 ********                    5 *************
 6 *********                   6 **************
 7 ******                      7 *********
 8 *******                     8 **********
 9 *****                       9 ********
10 *****                      10 ********
11 ***                        11 *****
12 *******                    12 ********
13 **                         13 ***
14 **                         14 ***
15 ****                       15 ***
16 ***                        16 **
17                            17
18 **                         18 ***
19                            19
20 **                         20 **
21 **                         21 *
22 *                          22
23 **                         23
24 ****                       24 ***
25 ***                        25 *
26 **                         26
27                            27 *
28                            28 *
29                            29
30 **                         30 **
31 **                         31
32 **                         32 *
33 *                          33
34 **                         34
35 **                         35 *
36 ***                        36 **
```

Table 3.1: Definition of Strategy 1, avoid self-recapture, and variable S1

If	S1 =
At least one alternative would result in a self-capture, and at least one alternative would not result in a self-capture, and a non-self-capturing choice was made.	+1
All choices would result in a self capture.	0
None of the alternatives would result in a self-capture.	0
At least one alternative would result in not self-capturing, and a self-capturing choice was made.	-1

Table 3.2: Definition of Strategy 2, attempt opponent-recapture, and variable S2

If	S2 =
Opponent capture is possible and opponent capture is chosen.	+1
Opponent capture is not possible.	0
Opponent capture is possible, but alternative chosen does not result in opponent capture.	-1

Table 3.3: Probability that position P can be recaptured by the opponent given that N turns remain to be played in Capture 1

P \ N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	.333	.555	.703	.802	.868	.912	.941	.960	.973	.982	.988	.992	.994	.996	.997
1	.611	.848	.941	.977	.991	.996	.998	.999	.999	.999	.999	.999	.999	.999	.999
2	.611	.848	.941	.977	.991	.996	.998	.999	.999	.999	.999	.999	.999	.999	.999
3	.555	.802	.912	.960	.982	.992	.996	.998	.999	.999	.999	.999	.999	.999	.999
4	.5	.75	.875	.937	.968	.984	.992	.996	.998	.999	.999	.999	.999	.999	.999
5	.444	.691	.828	.904	.947	.970	.983	.990	.994	.997	.998	.999	.999	.999	.999
6	.5	.75	.875	.937	.968	.984	.992	.996	.998	.999	.999	.999	.999	.999	.999
7	.333	.555	.703	.802	.868	.912	.941	.960	.973	.982	.988	.992	.994	.996	.997
8	.388	.626	.771	.860	.914	.947	.968	.980	.988	.992	.995	.997	.998	.998	.999
9	.277	.478	.623	.727	.803	.858	.897	.925	.946	.961	.972	.979	.985	.989	.992
10	.277	.478	.623	.727	.803	.858	.897	.925	.946	.961	.972	.979	.985	.989	.992
11	.166	.305	.421	.517	.598	.665	.720	.767	.806	.838	.865	.887	.906	.922	.935
12	.388	.626	.771	.860	.914	.947	.968	.980	.988	.992	.995	.997	.998	.998	.999
13	.111	.209	.297	.375	.445	.506	.561	.610	.653	.692	.726	.756	.783	.807	.829
14	.111	.209	.297	.375	.445	.506	.561	.610	.653	.692	.726	.756	.783	.807	.829
15	.222	.395	.529	.634	.715	.778	.827	.866	.895	.918	.936	.950	.961	.970	.976
16	.166	.305	.421	.517	.598	.665	.720	.767	.806	.838	.865	.887	.906	.922	.935
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	.111	.209	.297	.375	.445	.506	.561	.610	.653	.692	.726	.756	.783	.807	.829
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	.111	.209	.297	.375	.445	.506	.561	.610	.653	.692	.726	.756	.783	.807	.829
21	.111	.209	.297	.375	.445	.506	.561	.610	.653	.692	.726	.756	.783	.807	.829
22	.055	.108	.157	.204	.248	.290	.329	.366	.402	.435	.466	.496	.524	.550	.575
23	.111	.209	.297	.375	.445	.506	.561	.610	.653	.692	.726	.756	.783	.807	.829
24	.222	.395	.529	.634	.715	.778	.827	.866	.895	.918	.936	.950	.961	.970	.976
25	.166	.305	.421	.517	.598	.665	.720	.767	.806	.838	.865	.887	.906	.922	.935
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	.111	.209	.297	.375	.445	.506	.561	.610	.653	.692	.726	.756	.783	.807	.829
31	.111	.209	.297	.375	.445	.506	.561	.610	.653	.692	.726	.756	.783	.807	.829
32	.111	.209	.297	.375	.445	.506	.561	.610	.653	.692	.726	.756	.783	.807	.829
33	.055	.108	.157	.204	.248	.290	.329	.366	.402	.435	.466	.496	.524	.550	.575
34	.111	.209	.297	.375	.445	.506	.561	.610	.653	.692	.726	.756	.783	.807	.829
35	.111	.209	.297	.375	.445	.506	.561	.610	.653	.692	.726	.756	.783	.807	.829
36	.166	.305	.421	.517	.598	.665	.720	.767	.806	.838	.865	.887	.906	.922	.935

Table 3.4: Probability that position P can be recaptured by the opponent given that N turns remain to be played in Capture 2

P \ N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	.666	.888	.962	.987	.995	.998	.999	.999	.999	.999	.999	.999	.999	.999	.999
1	.444	.691	.828	.904	.947	.970	.983	.990	.994	.997	.998	.999	.999	.999	.999
2	.555	.802	.912	.960	.982	.992	.996	.998	.999	.999	.999	.999	.999	.999	.999
3	.611	.848	.941	.977	.991	.996	.998	.999	.999	.999	.999	.999	.999	.999	.999
4	.722	.922	.978	.994	.998	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
5	.722	.922	.978	.994	.998	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
6	.777	.950	.989	.997	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	1
7	.5	.75	.875	.937	.968	.984	.992	.996	.998	.999	.999	.999	.999	.999	.999
8	.555	.802	.912	.960	.982	.992	.996	.998	.999	.999	.999	.999	.999	.999	.999
9	.444	.691	.828	.904	.947	.970	.983	.990	.994	.997	.998	.999	.999	.999	.999
10	.444	.691	.828	.904	.947	.970	.983	.990	.994	.997	.998	.999	.999	.999	.999
11	.277	.478	.623	.727	.803	.858	.897	.925	.946	.961	.972	.979	.985	.989	.992
12	.444	.691	.828	.904	.947	.970	.983	.990	.994	.997	.998	.999	.999	.999	.999
13	.166	.305	.421	.517	.598	.665	.720	.767	.806	.838	.865	.887	.906	.922	.935
14	.166	.305	.421	.517	.598	.665	.720	.767	.806	.838	.865	.887	.906	.922	.935
15	.166	.305	.421	.517	.598	.665	.720	.767	.806	.838	.865	.887	.906	.922	.935
16	.111	.209	.297	.375	.445	.506	.561	.610	.653	.692	.726	.756	.783	.807	.829
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	.166	.305	.421	.517	.598	.665	.720	.767	.806	.838	.865	.887	.906	.922	.935
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	.111	.209	.297	.375	.445	.506	.561	.610	.653	.692	.726	.756	.783	.807	.829
21	.055	.108	.157	.204	.248	.290	.329	.366	.402	.435	.466	.496	.524	.550	.575
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	.166	.305	.421	.517	.598	.665	.720	.767	.806	.838	.865	.887	.906	.922	.935
25	.055	.108	.157	.204	.248	.290	.329	.366	.402	.435	.466	.496	.524	.550	.575
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	.055	.108	.157	.204	.248	.290	.329	.366	.402	.435	.466	.496	.524	.550	.575
28	.055	.108	.157	.204	.248	.290	.329	.366	.402	.435	.466	.496	.524	.550	.575
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	.111	.209	.297	.375	.445	.506	.561	.610	.653	.692	.726	.756	.783	.807	.829
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	.055	.108	.157	.204	.248	.290	.329	.366	.402	.435	.466	.496	.524	.550	.575
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	.055	.108	.157	.204	.248	.290	.329	.366	.402	.435	.466	.496	.524	.550	.575
36	.111	.209	.297	.375	.445	.506	.561	.610	.653	.692	.726	.756	.783	.807	.829

about 0.1 to about 0.9.

This situation suggests that in choosing a move a player should consider for each alternative the probability that the position could be recaptured by the opponent later in the game. Since these probabilities are based upon and positively correlated with the number of different ways in which an opponent could recapture the various positions, this strategy is called the minimum ways strategy. Strategy 3 and the associated variable S3 are defined in Table 3.5. Since Strategies 1 and 2 may take priority over this strategy, variable S3 has a rather complicated definition. These complications take into account the fact that a player may choose to ignore strategy 3, for example, in a case where there is one choice which results in recapturing an opponent but for which the number of ways is not minimal. Furthermore, Strategy 3 may be applied not to all of the choices but only to a subset of them, for example, in cases where a player uses Strategy 3 to choose among two or more alternatives which would result in opponent recapture, having decided to rule out moves that would entail self recapture or the capture of previously unoccupied places.

The fourth strategy is a strategy that combines features of the other three and gives priority to Strategies 1 and 2. The simplest way to define Strategy 4 involves first defining a function, Points (X), which assigns to each alternative X ($0 \leq X \leq 36$) a point value based on the number of ways in which position X can be recaptured and on whether it is currently held by the player, by the opponent, or by neither. The points function is defined in Figure 3.3; strategy 4

Table 3.5: Definition of Strategy 3, choose minimum ways, and variable S3

If	S3 =
Ways for chosen alternative is minimum of ways for all alternatives.	+1
More than one choice would result in opponent capture, and the alternative chosen has minimal ways as compared with ways for all opponent capturing choices.	+1
No opponent capture is possible, and more than one capture of a previously unoccupied position is possible, and the alternative chosen is a free capture with ways less than or equal to ways for all possible free captures.	+1
Exactly one opponent-capture is possible, and ways for that opponent capture is not minimal as compared with ways for all possible choices, and opponent capture is chosen.	0
No opponent capture is possible, exactly one capture of a previously unoccupied position is possible, ways for that position is not minimal as compared with ways for all possible choices, and the free capture alternative is chosen.	0
All possible moves result in self capture.	0
All cases not listed above.	-1

Figure 3.3: Definition of the function used to define strategy 4, coordinated strategy, in Capture 1 and Capture 2

If X is a board position $(0 < X < 36)$, then

$$\text{Points (X)} = \begin{cases} \text{Ways (X)} + 100 \text{ if X is held by opponent} \\ \text{Ways (X)} + 200 \text{ if X is unoccupied} \\ \text{Ways (x)} + 300 \text{ if X is held by player} \end{cases}$$

and the associated variable S4 are defined in Table 3.6.

The computer simulated player can be made to play using strategy 4 by means of a routine in the game program which selects Apple's moves by finding the alternative with minimal point value. In early trials of the experiment it was observed that children just learning the game were often quite badly beaten if Apple's moves were determined in this way. So it was decided that Apple's moves in the first few games with each child should use strategies 1 and 2 but not 3 and 4. This was accomplished by redefining Points (X) using a random number instead of Ways (X) as the first addend. Later in the interviews the children played several games against the computer-simulated player governed by strategy 4.

In cases where more than one alternative has minimal points, the minimal points alternative which is encountered first in searching through the alternatives is selected. In Capture 1 the order of search is sum, then difference, then product, then the smaller of the two place value alternatives, then the larger. For example, if both 23 and 32 were minimal points alternatives, 23 would be chosen. In Capture 2 the order of search is determined by the value of the player-supplied operand. For example, if 3 and '-' were rolled and the minimal points position were 3, then Apple would play 3 - 0 = 3 rather than 6 - 3 = 3 because the supplied operand 0 would be encountered before 6.

Qualitative Data -- Collection & Analysis

Qualitiative data have been gathered from the verbal interactions

Table 3.6: Definition of Strategy 4, coordinated strategy, and variable S4

If	S4 =
Point value of the alternative chosen is minimum of point values for all alternatives.	+1
All possible choices would result in a self-capture.	0
At least one non-self-capturing move is possible and point value of alternative chosen is not minimum of point values for all alternatives.	-1

that took place in the interviews. Audio tape recordings were transcribed and matched with computer generated records of the moves of the games so that references to particular situations in the games would be clear.

At the beginning of each interview the interviewer explained that what the child was thinking about as he or she played was of the greatest interest to him. Thus the children were encouraged to tell about their thinking if they could, and they were warned that the interviewer would ask them questions about their thinking at various times while they played.

A number of questions, which it was thought might be useful in the interviews, was prepared in advance. Those questions and lines of questioning, together with comments about their intended purposes and uses, are given in Table 3.7. In the actual interviews other questions and comments were also used, and no interview included all of the questions listed. It was never expected that an exhaustive and exclusive list of questions could be generated in advance of the interviews.

The analysis of qualitative data invloves comparisons of what was said with the moves that were actually made, as well as careful consideration of the words and phrases children have used in talking about their play and in answering questions. Attempts have been made to verify what these expressions mean to the children in order to gain insight into the children's thinking. It is recognized that what children say about their thinking and their actual thinking are not always clearly and directly related. The instances selected and

Table 3.7: Potential interview questions and comments on their use and purpose

Question/series of questions	Comments on use and purpose
What is the most points you could have at the end of a game? Do you always get this many? Why not?	Draws attention to 15 turns rule. Draws attention to possibility of recapture by opponent.
Does it matter which place you choose to capture? Why? What should a person look for or think about when choosing?	General question drawing attention to the need for a strategy.
What would be a good move? Why? What would be a bad move? Why?	General question drawing attention to components of (possibly subconscious) strategies. Perhaps most useful with reference to a particular move. May be followed up with questions about specific components mentioned by the player.
What happens to the score when you capture a place your opponent had? a place you already had? a place no one had captured yet?	Draws attention to the fact that opponent-captures are most desirable, free-captures are somewhat desirable, and self-captures are undesirable; leads to generalization of Strategy 1 and Strategy 2.

Table 3.7 (continued)

Are some numbers better to capture than others? Why? What are some good numbers? Why? What are some bad numbers? Why?	May draw attention to consideration of ease of recapture as a criterion; leads to construction and generalization of Strategy 3.
Do you think Apple has some particular way of choosing its moves? Can you predict what move it will make (in a specific situation)? Why do you think Apple chose that?	Draws attention to the fact that Apple uses a strategy rather than random choice; draws attention to components of Apple's strategy; encourages prediction and generalization.
Does it matter whether you go first or second? Why? Which player has the advantage?	Not directly relevant to counting or probability concepts, but does draw attention to strategy 2. Potentially useful in promoting generalization rather than reference only to particular situations and moves. Most likely used toward end of first game or later.
How are Capture 1 and Capture 2 alike? different? Does how you played Capture 1 help you decide how to play Capture 2? How?	Draws attention to similarities between games; suggests generalization/transfer of strategies and concepts.

reported in Chapter 4 are given with these reservations in mind.

Selection of Subjects

Over the course of more than two years the author has observed approximately 100 elementary school age children in various settings playing Capture in a variety of forms. Sometimes the observation has been casual and non-directive and aimed only at the collection of general impressions. At other times computer generated records and/or audio tape recordings have been made, carefully posed questions have been asked, and specific information has been gathered. These experiences have been quite valuable in developing and refining the routines and programs that are used to present the rules, display the game as it is played, choose the computer-simulated player's moves, and make records of each turn. Although a few examples from these preliminary experiences are mentioned in Chapter 4, that chapter is mainly concerned with findings from a carefully planned series of well documented interviews with six selected children that took place in May, 1983.

The main criteria for selecting children to serve as subjects for the study were the following. First, the study was restricted to children in grades 4, 5, and 6. Second, only children who had received no previous formal instruction in probability were included. Third, some familiarity with microcomputers was required (e. g. past experience using a microcomputer in school and familiarity with the Apple keyboard and the use of its 'shift' and 'return' keys). No specific experience was required; the purpose of this criterion was only to exclude children who would be awed or intimidated by the novelty of using a computer.

The six students whose performances are reported in detail in Chapter 4 were students at University Elementary School (UES) in Calgary, Alberta, Canada. They were all students in a single Division II (i. e. grades 4, 5, and 6) block, a multiage unit under the direction of two teachers in an area about the size of two ordinary classrooms. This school was chosen primarily because of its accessibility to the researcher who has served on one of the school's advisory committees and directed in-service activities for teachers there. Because UES serves as a demonstration school for both the University of Calgary and the Calgary Board of Education (the latter is responsible for its administration), its teachers are carefully selected and are paid an additional stipend beyond their normal salary. The school is located in Northwest Calgary near the University of Calgary and the Foothills Hospital. About half of the students attending UES live in the immediate vicinity of the school; many of them are the children of professionals employed at these two institutions. Two other middle class or upper middle class neighborhoods are also in the catchment area, but students living in these areas come to school by bus. About one-sixth of the students are accepted (on a first come first served basis) from waiting lists of children living outside the catchment area. The school is quite favorably regarded by the public, and the waiting lists are so long that it usually takes more than a year to reach the top of the list.

The six children who took part in the interviews were selected by their teachers who were asked to choose students of both sexes, of all three grades, and of varying ability. All the children in the block met the computer familiarity criterion, since Apple microcomputers have been in use for several years at UES, and the children have done simple word processing, programming in BASIC, programming in Logo, and other computer activities at various times in the past. None of the children had recieved formal instruction in probability. The sex, age, and grade of each of the six children is shown in Table 3.8. Pseudonyms have been used to preserve the anonymity of the subjects.

Although it is clearly impossible to claim that the subjects in this study were randomly selected as a representative sample of the universe of all students in grades 4, 5, and 6, the children did display a range of behaviors and ability levels. Since the purpose of this study is to explore the effects of using Capture in a particular way, and since its aim is to gain insight into this process rather than to make generalizations about populations of children, the representativeness of the sample is not a central issue. The disadvantages inherent in using a restricted sample have been weighed against the costs that would be involved in collecting and analyzing data from a broader field, and a judgement has been made that this sample is adequate for its purposes.

Table 3.8: Name, sex, age, and grade of subjects

Name	Sex	Age (years: months)	Grade
Alan	M	10:2	4
Beth	F	10:1	4
Carl	M	10:1	5
Dawn	F	10:11	5
Elsa	F	11:2	6
Fred	M	12:0	6

CHAPTER 4

FINDINGS

In Chapter 1, the setting and significance of the problem under investigation in this study were described, and the problem was specified in terms of seven major questions to be answered. In Chapter 3 the methods to be used in answering those questions were outlined. In this chapter the findings are presented in the following manner. First, the results of the experiment are presented. Some general observations from the interviews are given followed by description, analyses, and selected excerpts from the interviews with each child. This is done in order to provide an overview of the results in general and insights into particular instances which bear on the study's major questions on an individual basis. In the second part of the chapter each of the seven major questions is considered in turn, and answers that reflect the experiment as a whole are offered.

Experimental Observations

General Observations

The interviews were conducted as planned, and no hardware or software problems were encountered. The children seemed to have little difficulty reading the rules and instructions or interpreting the graphic displays. Occasionally a child asked the interviewer a question that was already answered by information on the screen display. In these cases the interviewer usually pointed out where the answer was to be found (e. g. "It's Apple's turn" and "Press any key to go on"), and the problem was resolved. Sometimes the children seemed to be seeking reassurance rather than an answer to a question. In playing their first moves in Capture 2, some children made several attempts before satisfying Apple's requirements for a number sentence. In these cases the prompts provided by the game's input processing routines (e. g. "Use '=' in your sentence" and "Complete the sentence") seemed adequate in informing the children what was needed. Three of the children asked how they should indicate multiplication in their input in Capture 2, but the others assumed that it was to be indicated by '*.' It is fortunate that the program was written to accept either '*' or 'X.'

In the following sections, the focus is mainly on the analysis of qualitative data, and quantitative data are used as appropriate to support these analyses. Complete summaries of the quantitative data are given in tabular form in Appendix 2. In these tables the values for S1, S2, S3, and S4 are given for each child's moves in each game on each day, together with the numbers of times each variable was positive, negative, and zero in each game.

As was noted in Chapter 3, the subjects were selected by their teachers as a sample of the students in their block that would represent, as well as it could, the range of the children's ages, grades, abilities, and personalities. The interviews were conducted by the interviewer before he had any information about the individuals with respect to these characteristics. After the interviews were completed, the interviewer met with the teachers to share with them some of his observations and to collect from them some background information on each child. In retrospect that information has seemed helpful in understanding some of the children's reactions in the interviews, so it has been taken into account in the presentations which follow, even though it was not available to the interviewer at the time of the interviews.

Alan

Alan is a quiet grade 4 student whose teacher claimed she often has to "pry" to get him to respond verbally to her questions in class. His level of achievement is generally good, but he seems to work only just hard enough to satisfy his parents' and his teachers' expectations as he perceives them. He enjoys reading, but is reluctant to write; on standardized achievement tests he scores somewhat above grade level in reading comprehension, but below grade level in spelling. In mathematics his test scores are close to grade level. Earlier in the year he showed weakness in basic facts, but with encouragement and extra drills monitored by his parents and his teachers he improved noticeably; when the extra work was later relaxed, his skills seemed to decline again. He has shown no special interest in or talent for problem solving, and in this sort of work he often prefers using manipulative materials. Making records of his concrete work is something he seldom does except when urged to do so. There is an Apple microcomputer in his home which his father uses for word processing and which Alan uses to play arcade games. He has been taught some BASIC in school, but he has done little more with it on his own.

In the first day's interviews Alan displayed the same reticence that his teacher described as typical. He did not respond to general requests to think aloud, and his answers to specific questions were often mainly umm's and ah's. Early in the first game he described his thinking as "trying to close in on" Apple's markers. His answers to specific questions about the current score and about his predictions of changes in the score that would result from various moves were unclear, but they did seem to suggest that he understood the value of recapturing his opponent's places. On his last three turns of the game, however, Alan did not recapture when he could, so at the end of the game the interviewer initiated the following exchange.

I: You had 15 turns and Apple had 15 turns, right?

A: Right.

I: How come Apple ended up with only 11 places and how come you ended up with only nine?

A: Well, if you land on a place that the other person has taken then you take away one point and gain one point. So, ..., umm, ..., he landed on me ... umm ... I don't know how many times, and he took away so many points.

It appears that these questions caused Alan to realize the purpose of trying to land on the opponent's markers, for in game 2 he used strategies 1 and 2 at every opportunity except for one turn in which recaptured his own position, apparently mistaking it for one of Apple's. At the end of the second game the interviewer observed that Alan must have learned something, since he lost the first game but won the second. Alan's reply confirmed the hypothesis.

A: In the first game I thought there'd be one number there and you had to get each one of yours on the outside of them. Near the end of the game I realized you don't try to get that, you try to get the other person's, you try to get right on top of theirs.

On day 2 Alan used strategies 1 and 2 consistently. At turn 3 of game 1 and again at turns 4 and 8 he reported using a strategy that has some similarity in its logic to that of strategy 3. Alan observed that when he tried to place his markers near Apple's markers, Apple often recaptured them later on. So, he reasoned, he should place his markers farther away so that Apple might not recapture them.

At the end of game 1 the interviewer observed that certain numbers seemed to change hands repeatedly, and he asked Alan what numbers he thought did that "a lot." Alan would not venture an answer, but he did suggest that "if you played the game a lot, you could figure it out." When asked to answer on the basis of his own experience, Alan suggested "24, and ... uh ... six or seven." In actuality 24 changed hands twice, but six and seven were never captured, not even once. He failed to mention 10, which changed hands four times and four and eight which did so three times each. When asked for a number which did not change hands often he suggested 20. Alan seemed not to have much confidence in his predictions; nor did he seem to recognize that choosing numbers on the basis of whether they changed hands frequently or infrequently might be a good idea. At turn 8 in game 2 Alan was asked to consider each of the alternatives and tell whether any of them seemed to change hands frequently. His reply was given with reluctance and great uncertainty, as was his reply to a question asking him to predict whether each place would be recaptured if he chose it.

At the end of game 2 the interviewer asked a series of leading questions about the different dice rolls that would be needed in order to capture each of the alternatives in turn 8, which was discussed before. As Alan haltingly listed the combinations, the interviewer wrote them down. When the chart was complete he showed it to Alan, and carefully made observations in the follwing forms: 'in order to capture this position Apple would need to roll this or this' and 'in order for Apple to capture this he could roll any one of these.' Alan showed no sign that he thought such information had any value. (This is in marked contrast to other children, such as Carl, who readily concluded that the "harder to make" positions were more desirable.) These results indicate that while Alan indicated he wanted to choose positions that Apple might not recapture, he had no way of predicting which positions those might be. There was no evidence that he understood strategy 3 based on observed (empirical) or predicted (theoretical) probability.

In playing Capture 2 on day 3 Alan's performance was similar to his playing of Capture 1. He always used strategy 1 and almost always used strategy 2, the exceptions being cases where he overlooked an opportunity to recapture. Although he answered the question "Do you think anything you learned playing Capture 1 helped you in deciding how to play Capture 2?" in the negative, he described his strategies in each case in almost identical terms. When asked which places seemed to change hands most often, he named four of the six most frequent, but he failed to mention the position that was recaptured the most frequently, position 1. Although Alan seemed to have a better idea of the actual frequencies of recapture, there is no evidence that he applied this information. When asked why he thought those places changed hands most often, he could offer no reason.

Beth

Beth is a grade four student who transferred to UES at the beginning of the year. An outgoing, happy, and self-confident child with a good sense of humor, she seemed to find the transition easy. Beth has good work habits, and her written work is usually neat and carefully proofread. In problem solving she shows persistence and willingness to work on her own, and when she is stuck she can usually tell what she has tried when asking for help. She shows flexibility in using manipulatives, tables and charts, and patterns and formulas. Her standardized achievement test scores are near grade level, with language areas stronger than mathematics. Beth has played educational games (e. g. Hangman) and arcade games in computer clubs held outside of regular school hours, but she has had only limited experience programming in BASIC.

On day 1 game 1 began with a series of captures and recaptures. By the fourth turn each of the players had recaptured the opponent's position twice each, but it was not clear whether Beth appreciated what was happening. At turn 9 Apple was winning by a score of seven to three, and the interviewer asked Beth why. In reply she observed, "I guess I didn't really get the game at first, ... and then I started realizing how the game was played." But when she was asked specific questions about what would happen to the score if she chose various self-capturing and opponent capturing moves, it was clear that she did not understand. She referred to an opponent held position as "already used up," and hypothesized that if she chose that value it "might move down." She also thought she "might get an extra point" if she chose a position she already held. The interviewer suggested that she should try choosing the value of a place already held by Apple and observe the result. This she did, and when asked she agreed that that was a good move. Thereafter she always looked for ways to capture Apple's positions and consistently used strategies 1 and 2.

In game 2 on day 1 she was asked about how she chose among possible moves when all of her alternatives were free captures. On several turns she listed the possibilities and commented that Apple might later recapture them. She often mentioned particular dice outcomes which would permit this, but never did she indicate that she thought any of these events was any more or less likely than the others. All of her predictions were given in terms of what "could" or "might" happen. At turn 9 of game 1 Beth and the interviewer again discussed her reasons for choosing among free capturing alternatives.

B: I guess I'll choose nine.

I: Is there any special reason for choosing nine?

B: Well, ... maybe a low number because Apple might go for a higher number.

I: So did you choose that so that Apple won't take away your place?

B: Well, you really don't know what Apple will choose next.

The investigator acknowledged the uncertainty of the situation and pointed out that even with uncertainty it is sometimes possible to predict that some outcomes are more likely than others. He used weather prediction as an example and continued.

I: Is there any way to predict that it's more likely for some places to be taken back from you than others?

B: No, ... well ... No, you really don't know what Apple is going to do.

Later, at turn 1 of game 2 Beth chose to capture 1 and commented as follows.

B: Maybe if I took that one, ... I'd have a better chance ...

I: A better chance of what?

B: Of maybe not losing it ... (chuckles) ... but I think I am going to lose it.

In fact, Apple did recapture the position on the very next turn, so Beth's very tentative venture into choosing positions that might not be recaptured met with failure. She never seemed to pursue this tactic again.

On day 3 Beth played Capture 2 using strategies 1 and 2 as she had in Capture 1. She made two self-capturing moves in game 1, but she immediately recognized these errors and explained them on the basis of forgetting or of mistaking one mark for another. She also failed to capture her opponent's marks several times when she could have; some of these missed opportunities she noticed herself, and some she recognized only when questioned. When she was asked which version of capture was harder, she felt there was "more to think about" in Capture 2.

Carl

Carl is one of the youngest children in grade 5; at the time of the interviews he was 10 years and 1 month old. Carl has a reputation among the students and teachers at UES as an excellent problem solver and computer programmer. As a project in grade 4 he wrote a program using Applesoft graphics commands to make two jousters appear on the screen, gallop toward each other, and crash and explode when they met. At about the same time he learned to solve the Rubik's cube puzzle, and before long he had children from across the school coming to him for help with their cubes. In grade 5 his science fair project was an Applesoft program using mixed text and graphics showing the movement of stars in the night sky. Other programs he has written include a program that produces the elements of Pascal's triangle and one that produces prime numbers. For several years Carl's family has had a Pet microcomputer which he uses mostly for playing games but also for writing programs. Carl explained that sometimes he takes the listings of game programs that are published in microcomputer magazines and tries them out, learning about computer programming in the process.

Although Carl is clearly an exceptionally good problem solver, standardized achievement tests put his arithmetic computational skills at about grade level. His spelling is below grade level, and his handwriting is poor, but his other verbal skills are close to grade level expectations. Carl is a pleasant child and he approached the interviews with interest, confidence, and politeness.

At turn 2 in game 1 Carl had just chosen B, when he was asked to tell what he was thinking about.

C: Well, I was thinking about the answer for all of them.

I: Uh huh, and did you work out each one?

C: Yep, and then I found out ... I'll probably go to C next and try them all out.

At this point the interviewer had not noticed that Carl had played A on turn 1 and B on turn 2, nor had he suspected a pattern in Carl's play. The comment revealed, however, that Carl had a plan that he was using to guide his initial trials. The interview continued:

I: What would have happened if you had chosen something else, other than B? Does it make any difference which one you choose?

C: No, not really.

I: What would happen if you chose A (a self-capturing alternative)?

C: If I choose A, I'll capture 11, and I already have 11.

I: So you decided to do B instead?

C: Uh huh.

Thus strategy 1, avoid self-capture, was immediately recognized as obvious. Carl used strategy 1 at every opportunity on all three interview days.

At turn 4 of game 1 Apple captured one of Carl's positions, the first possible recapture of the game.

C: Hmmm.

I: What did you just notice there?

C: He took one of mine.

Two turns later Carl had his first opportunity to recapture.

C: Hmmm.

I: What do you think?

C: (Chooses A, the opponent capture, but does not comment.)

I: Why is that a good move, Carl?

C: I just took mine back (recaptured from Apple a position that was captured first by Carl then immediately recaptured by Apple).

I: Is that a good thing to do?

C: Yeah, then I'll have one more place.

I: Yeah, but you'd have one more place if you just took any one, wouldn't you?

C: No, ... ,yes, but I took his so he'll have one less, too.

I: So it's better ...

C: (Interrupting) It's better to take one of his than one of the others.

Thus, strategy 2, attempt opponent capture, also seemed obvious to Carl, and he was able to explain why it was valuable. Carl used this strategy when playing Capture 1 throughout days 1 and 2, but when playing Capture 2 on day 3 Carl omitted to use strategy 2 on several moves because he had not recognized the possibility of capturing his opponent's 0 when he rolled multiplication.

The first evidence that Carl was beginning to construct strategy 3, choose minimum ways, appeared at turn 8 of game 1 on day 1.

I: Carl, tell me what you're thinking about now. First, what one are you going to take?

C: D, because it's right there (pointing to 33), and I don't think he'll have another question that will go as high as that.

I: And you think it's better to take a high number?

C: Yeah, well ... , yeah, I don't think he'll have that exact same one again.

At this stage, Carl expressed this strategy in terms of desirable numbers being high numbers, in particular those which can be made only with the place value alternatives, and he supported this with the argument that a specific pair of numbers must be rolled to get them.

There are several cases where Carl was thinking in terms of strategy 3, but failed to apply it correctly; all involve an incorrect estimate of the probability of being able to capture position 0. At turn 9 of game 2 on day 1 Carl rolled a pair of threes, and none of the four alternatives allowed opponent capture or self-capture.

C: I'll go for 0; it's not often you get a 0.

I: You were saying it's not often you get a 0?

C: Yeah, and since you don't get it often, there's less of a possibility of him taking 0.

I: And of all the possible ones you had to choose do you think that's the one Apple would have the hardest time to get?

C: Yep. ... 33 might be quite hard to get, too.

I: But you chose 0 instead of 33?

C: Yeah.

In the 47 turns that had been played up to that point, doubles had occurred only twice and neither player had yet chosen 0. So, Carl's observation was correct in an experimental probability sense. On day 2 Carl also chose 0 instead of a higher number. On turn 1 of game 1 he chose 0 instead of 25, and explained "you have to get a double number to get 0, like 2, 2 or 5, 5, so that lowers the possibility of him getting 0. Then he can't get mine unless he gets 0." At turn 5 of game 2 Carl chose 0 over 22 and was asked to explain why.

C: Because you have to get a double number, and the other number, the higher number was 22, and I think it will be harder to get 0 than 22.

I: You think it will be harder to get 0 than 22?

C: I'm not sure of that, but I think ...

I: Is there any way you could find out? Whether ...

C: Yeah you could find all the facts of 22 from six and all the facts for 0.

I: OK, maybe we should do that. (getting paper)

C: (Without hesitation) There are six ways to get 0. (They list them together, and the interviewer writes them down.)

I: What about for 22?

C: Two and two. ... (A long pause during which Carl whispers inaudibly, apparently saying number facts) ... and that's all!

I: So what does that tell you?

C: You get 0 more than 22.

Several facts should be noted about these exchanges. First, Carl had a clear intuition that doubles are rare and that for that reason 0, which can only be made with doubles, is a good choice. But when pressed to say whether C is better than other numbers which can be captured with doubles, Carl responded with uncertainty. When asked for a way of resolving the uncertainty, Carl immediately suggested listing the possibilities, and he immediately recognized that there would be exactly six ways to get 0. Taken together these facts suggest that the intuition of the rarity of doubles was being used to select a move, but not a logical analysis, even though Carl was readily able to use that sort of formal reasoning. In the comment "you get 0 more than 22" Carl did not seem to be discriminating between expected (theoretical) probability and actual (observed) probability.

Aspects of strategy 4, the coordinated strategy, were also in evidence. The following exchange which took place at turn 10 of game 1 shows that Carl had decided to subordinate strategy 3 to strategy 2.

I: What were you thinking here? You looked through all of these (possibilities) to see what number they were?

C: Yeah, and C, I picked, because it's 15 and he has it, and then I would capture his, and then he would have one less, and I would have one more.

I: OK, well the last time you played, just a turn before, you told me that you liked 33 ...

C: (Interrupting) because it's a high number ...

I: Because it's a high number, and you could have chosen D (35) which would give you another high number. But you decided not to?

C: Yeah, it's better to get, to take one of his so he loses and I get a point. But here (35) I'd just get one.

The hierarchical analysis that Carl uses is clearly seen in his comments about his choosing for turn 12 of game 1.

I: Tell me what you're thinking about here.

C: Well, I can't take one of his, and I already own 15, so I can't really, so D's not such a good choice (15). Five I already have, so C's eliminated, and these two are my two choices, (pointing to four and six) right here and right here.

I: So, do you think it makes any difference whether you choose 5 + 1 or 5 - 1?

C: No. (Chooses A)

In fact, it does not make a difference, because the numbers of ways for four and for six are equal. But there is no evidence that Carl knows this fact; on the contrary, it appears that he is just choosing the higher number. Perhaps, since neither of the two numbers is particularly large and both seem reasonably likely to be recaptured, Carl did not see any point choosing between them.

Carl seemed to understand the need for and value of a strategy, but he did not use the terminology of probability or expectation to express this idea. At turn 8 of game 2 on day 1 with the score tied at 5 to 5 Carl was asked whether he thought he was going to win.

C: Well, I, ... The last time I started, I didn't really have a strategy, and, ... but, ... and I tied. So this time I could win.

Even though his prediction was expressed only in terms of the possiblitiy of winning, the fact that he compared and contrasted the previous game played without a strategy and the present game played with a strategy suggests that he believed it was now more likely that he would win. This is just one of several instances which show that Carl was thinking about comparing the likelihoods of outcomes, even though he did not express the idea using the terminology of probability.

In playing Capture 2 Carl used strategies similar to his strategies for Capture 1. He explained his version of strategy 3 again in terms of wanting to take large numbers, and when questioned why these were preferred, he correctly listed their factors and contrasted the few ways of getting them with multiplication with the large number of ways of getting small numbers using all three operations. Carl tied the first game six to six and won the second nine to five. In the first game all of his moves were optimal (i. e. consistent with strategy 4), but in the second game he made eight suboptimal moves, failing to use strategy 2 four times and strategy 3 six times. These results underscore the fact that the four variables by themselves are not reliable indicators of the children's understanding. To have used scores in the games as indicators would have been even more misleading.

Dawn

Dawn is an attractive fifth grade girl with a sunny personality and a ready smile. She is considered quite intelligent, but she sometimes seems not to put enough effort into her school work. Earlier in the year her younger brother was promoted (i. e. he skipped a grade), and Dawn seemed to react by slacking off. When her parents and teachers met to discuss this, they decided to let Dawn know that they considered her to be bright and to be capable of much better work. This approach seems to have been effective, for Dawn's achievement since then has been good. Dawn is strong in language arts, and her mathematics is good to above average, but she has shown no particular interest in arithmetic, computer programming, or problem solving. Dawn's science fair project this year was on the relationship between barometric pressure and arthritis pain. Her mother was her willing subject.

In the interviews Dawn was friendly and moderately talkative. An indication of an early, spontaneous strategy can be seen in the following comments Dawn made when she was asked what she was thinking about in choosing her move for turn 2 of game 1 on day 1.

D: I was deciding whether I should put it beside one of the A's or just somewhere else.

I: Do you think it makes a difference if you do that?

D: Probably not.

At turn 4 of game 1 Apple recaptured a position Dawn held. Her nonverbal reaction indicated she had noticed something, and when questioned she explained "He took my spot" ... "by choosing one I already had." Thereafter she used strategies 1 and 2 consistently, except for one time when she recaptured one of her own positions by mistake.

Dawn did not seem to understand strategy 3, and the values of S3 show that her play in both games on day 1 was much more often inconsistent with this strategy than consistent with it. The following exchange which took place just after the first game ended indicates that she understood a part of the rationale for strategy 3, but only a part.

I: I noticed that sometimes when you were playing you seemed angry at the Apple. Why is that?

D: It's frustrating. ...

I: Why is it frustrating?

D: Well like that one time when he just kept taking them over, ... and afterwards I noticed I maybe should have just got onto something else 'cause I might have gotten more.

I: Gotten more? ... If you did what?

D: If I hadn't gone for the same places.

I: Well, do you think that some places are better to take than others?

D: Well, it depends ... 'cause if the A's there that may be better.

From this point in the conversation Dawn went on to describe strategies 1 and 2. These are essentially offensive strategies, but the defensive strategy of choosing positions that would be harder for the opponent to recapture did not seem to have ocurred to her.

While Dawn was playing Capture 1 on day 2 the interviewer interrupted her at different points to ask several series of questions that would check her understanding of (or perhaps lead her to an understanding of) strategy 3. In each case three or more free captures but no opponent captures were possible, and the questioning always had to do with which alternative was best. Dawn was asked to predict whether Apple would be able to recapture each of the places if she took them, and to tell whether it was more likely that Apple would recapture some of them than others. She was asked whether any of the places seemed to change hands repeatedly and whether any of them tended to be taken and not recaptured once taken. She was also asked whether she thought Apple might find it harder to recapture some of the positions than others. In one series of questions she was asked about the different dice rolls that could be used to capture the positions and about how many different dice outcomes there were for each. Finally, she was asked whether these questions might help her to defeat Apple if she thought about them in a future game. In each of these variations of the questioning her answers were consistently tentative; they were often qualified with phrases such as "you don't know for sure, but it seems." Many of her estimates and predictions were reasonable, but she did not seem to believe in them very strongly. The interviewer felt that for Dawn the questions were mere speculations and not a kind of reasoning she was comfortable using.

On day 3 Dawn played Capture 2 in much the same way she had played Capture 1 on the previous days. She used strategies 1 and 2 in all but two cases which were probably oversights, but in one case she chose to capture 36 which was unoccupied, so it may be that she was using a form of strategy 3 without subordinating it to strategy 2. Her understanding of strategy 3 seemed to be growing, even though variable S3 was negative five times in game 1 and eight times in game 2. At turn 12 of game 2 Dawn chose to capture 36 and justified her choice with the comments, "I doubt Apple will go for it because he usually goes for lower numbers," and "I thought it would be the safest." When generalizing about how she decides what move to choose, she immediately described strategies 1 and 2, then added that she looks for numbers that Apple might not get away from her. Asked to characterize these, she mentioned only "numbers past the 30's." In her answer to the question 'what did you learn in playing Capture 1 that helped you to play Capture 2?' she mentioned strategies 1 and 2 but not 3. On the whole it seems that she had some idea of the purpose of strategy 3, but she did not use it consistently or explain it clearly or completely.

Elsa

Elsa is an extremely intelligent and extremely quiet sixth grade girl. Although she often speaks with hesitation, her words are clear and usually well chosen. Her verbal skills, and in particular her reading comprehension, are far above grade level expectations. She enjoys reading fantasy authors such as Tolkien, and her written work often shows both creativity and attention to detail. One notable recent project was a newspaper in elf-script which was meticulously and artistically done. Her mathematics achievement is substantially above grade level, and although she has no particular difficulties with mathematics, at the same time she also does not seem to have any special interest or talent in the area either. Elsa's grade five mathematics class included an extended unit on problem solving with computers using BASIC in which the children learned flowcharting, the use of variables, and debugging in the process of solving problems on number patterns. When asked whether she had used the computer much, she replied, "No, not much, except for word processing."

In the interviews Elsa was often reluctant to verbalize her thinking, but on several occasions she gave retrospective reports that were clear and incisive. On each of her first two turns she was asked to tell why she chose as she did, but she appeared to be unable to do so. Finally she remarked that she chose 11 on turn 2 "because there's something about the number 11, ... I like it." The fact that she took a fairly long time to decide suggested to the interviewer that there was probably much more to it than that. Although the values of S1 and S2 were negative once and five times, respectively, in the first game, Elsa gave clear explanations of both strategy 1 and strategy 2 in answer to a series of questions at the end of game 1. "First I look to see if there are any I can use to wipe out the A," she said and explained that "It wouldn't be any use to pick another place that I've already gotten, and just waste a turn." Nonetheless, her scores on S1 and S2 were also negative several times in game two. But again she had clear explanations of what her difficulty had been. On several turns in succession she tried to recapture the 1 that was held by Apple, which was next to the character 'I' that was used to mark 0 on the number line. Mistaking the "I" for 1, she chose 2 each time, missing her target and recapturing her own position.

On day 2 Elsa used strategies 1 and 2 throughout both games, and she developed strategy 3 along the way. In game 1 S3 was negative five times and positive four times; in game 2 S3 was negative only twice and positive eight times. Several times during game 1 remarks were made (by the interviewer and by Elsa) about the fact that certain positions changed hands frequently, while others did not. When she was asked to tell what numbers belonged to the latter group, she replied, "Well, there's 36, ... , they're mostly above the 20's, or above 15." This is a remarkably insightful characterization (cf. Figure 3.2). When she was asked why this was so, she observed, "Well, the dice numbers don't go very high, ... and unless you make a choice like 3 tens and 5 ones, then you usually won't get in the high numbers very much."

Between the two games Elsa was asked whether she thought she would win the next game. When she replied that she did not know, she was asked by the interviewer to "predict what the chances would be." This was the first time that the term 'chance' or 'chances' was used, but not the last, for Elsa used these terms repeatedly in subsequent exchanges. At turn 1 of game 2 there was extended discussion in which Elsa observed, among other things, that if she chose a low number Apple would "have a better chance of taking it, whereas if I chose 33, it's less frequent." The discussion also included consideration of the dice roll outcomes that would be needed for a player, in particular Apple, to recapture each position. Although Elsa showed some hesitation in listing all the possibilities, with prompting she found them all, and she concluded from the discussion that 33 would be the best choice because "there's less chances" of it being recaptured. It was not clear whether she was using this phrase in the sense of probability or in the sense of each of the ways (i. e. dice roll outcomes) being a 'chance' to capture the position.

On turn 8 of game 2 Elsa rolled a two and a three, and Apple held both 5 and 23 (choices A and D). Elsa explained that she thought D was better "because 23 is a higher number, and 5 is one of the more common ones, ... so I prefer 23 ... because there are less chances of getting number 23." This statement shows good understanding of strategy 3, and it seems to be based on the generalization that the higher numbers "have less chances" (i. e. there are fewer ways of making them) and on the observation that five is "common" (i. e. it occurs frequently). Elsa seemed to be combining empirical probability reasoning and theoretical probability reasoning in the same argument. She was also coordinating strategy 2 with strategy 3, in that strategy 3 was being applied to a subset of the choices because strategy 2 applied. That is, she used strategy 4.

On day 3 Elsa played Capture 2. On turn 2 of game 1 she was asked to explain her thinking.

E: Well, it's almost the same as yesterday, but I had to add, so I chose the highest number.

I: Why the highest?

E: Because the Apple probably wouldn't get, ... probably wouldn't have as much chance of getting the same one, ...same things on the dice.

This argument is expressed less clearly than others of Elsa's arguments, and it shows an interesting shift from prediction, "Apple probably wouldn't get," to comparison of predictions expressed using the idea of chance, "wouldn't have as much chance of getting." Later, when discussing game two, Elsa was more precise, and she used the term 'less likely.' She observed that she prefers to choose "a number that is higher up, ... higher than 10, ... because Apple is less likely to get a selection that could take away those places."

From her comments it is clear that Elsa generalized her arguments (e. g. about choosing higher numbers, about predicting chances of outcomes), but she also noted some differences between the games that resulted because the operation was fixed. Elsa thought Capture 2 was the more difficult, and she pointed to the greater number of possibilities a player has to consider.

Fred

Fred is a grade six boy who has well above average intelligence and very good social skills. Even though he has excellent general ability, these traits are not always seen in school because of his "laid back" attitude. Fred tends to follow the line of least resistance, and he often does as little work as is required to get by. His performance on standardized achievement tests in mathematics is substantially above average, and Fred often shows good understanding, exhibiting what Krutetskii (1976) referred to as "curtailed reasoning." Fred is very interested in computer programming, and his family has an Apple at home on which he is learning Pascal. Fred commented that he was not interested in computer games, but his teachers reported that he has access to a large amount of software. As a programmer in BASIC he is very good, but he seldom shows the clear organization and the finesse that are typical of Carl's solutions. His programming in Logo has also been somewhat advanced, and he has exhibited a tendency toward structured thinking to a greater extent than his classmates.

In game 1 on day 1 four of Fred's first five turns were ones in which an opponent capture was possible. Apple, on the other hand, did not have the possibility of capturing one of Fred's places until turn 6. Although Fred mentioned both "replacing" one of Apple's men with his own and going as far away as possible from Apple's markers as two possibilities he might use as a strategy, he did not actually choose an opponent capturing move until turn 6, after Apple had first recaptured one of Fred's positions. Once he saw this strategy in action, he immediately tried it for himself, and he continued to use it on seven turns in game 1. He never recaptured one of his own positions except when he mistakenly "chose the wrong one." When asked to justify these strategies, Fred pointed to changes in the score as his reasons, and correctly described what would happen to the score in each type of situation. When Fred was asked at turn 5 in game 1 to predict the outcome of the game, he predicted either the Apple winning or a tie. When asked if he thought he "could win," he replied, "It's not likely." Fred's moves sometimes but not always seemed to be consistent with strategy 3, so he was asked how he was choosing his moves.

F: Well, I'm going to take a high one.

I: Why is it better to take a high number?

F: It doesn't mean anything really.

I: Do you think it is harder for Apple to take away a high number?

F: Not necessarily.

On the basis of responses such as these, the interviewer concluded that Fred was simply considering alternate strategies, and did not have any clear idea of strategy 3.

As the interviews began on day 2, the interviewer again asked Fred to think aloud, likening what he wanted to talking to oneself and reminding Fred that the purpose was for the interviewer to be able to know what he was thinking about. Fred's verbalizations were quite detailed throughout both games that day. In game 1 ten of the 15 turns permitted Fred to recapture one of Apple's positions. On these turns Fred explained that he was looking for ways of taking away one of the A's, and he did so each time. On the remaining turns, where opponent capture was not possible, Fred observed that the Apple hadn't given him "much of a choice," and on two of these turns Fred remarked, "I'm going to go on to another one and hope that he doesn't take it back." Although these comments revealed that Fred had recognized the need to forsee the possibility of recapture by the opponent, strategy 3 was still not well formed, as is shown by the fact that Fred offered no comment on either occasion when he was asked the following two questions: "Is there anything you could do to have a better chance that he wouldn't take it away from you?" and "Do you think it might make any difference if you took one or the other, about whether the Apple might be able to take it back from you in the future?"

At the end of the first game Fred and the interviewer had the following discussion.

I: Have you noticed that some places on the board seem to go back and forth, ... first you take it, then the Apple takes it away from you, then you take it back again, and so on?

F: Yes. Yesterday I noticed that also, so today I was choosing other places.

I: Other places? ... What was your reason for that?

F: Well number five, last time I chose it, it kept on going back and forth so this time I chose four or six and he chose five.

I: And by choosing something else ... ?

F: He didn't mind.

I: You mean he wouldn't take it back from you?

F: Right.

I: Are there some numbers on the number line that don't go back and forth like that so much?

F: Oh yes, quite a lot. ...

I: Like what? ... What kind of numbers don't get traded back and forth so much?

F: Well, other high ones above 25, they don't, and ... umm ...

I: Why do you think that is, that the high numbers don't go back and forth so much, but the low numbers do?

F: Maybe the computer's programmed to only take the low numbers.

I: That could be, ... I can tell you something about the way the computer is programmed. It's programmed so that the dice come up at random, ... it uses a random number generator.

F: Right.

I: Do you know about those?

F: Yeah.

I: So that it isn't trying to get the low numbers, it's just using whatever it can with the dice.

F: Uh huh, right ... So with the dice it will take any of mine if it can.

I: It tries to take them away from you if it can, and it tries to keep from taking ones that it already has.

From this segment of conversation it is difficult to be certain what Fred "really" understands and what he was simply willing to agree to. However, certain facets of it may be worth noting. First, Fred has apparently been noting the numbers which change hands often. His suggestion that five changed hands often but four and six did not, is not an accurate characterization of the game he just finished, nor is it expected on the basis of the theoretical probabilities, but his observation that numbers above 25 do not change hands often is a valid one. Fred's view of what the computer program is "trying to do" is not clear, but it is clear that he understands strategies 1 and 2, and it seems likely that he understands that Apple uses them, too.

In game 2 at turn 4 another discussion took place. It seemed to show that Fred understood strategy 3, but it also included prompting by the interviewer. Fred had rolled a six and a four; he already held 10, and the discussion was about whether it would be better to choose two or 24.

I: From what you were telling me before, do you think it would matter whether you took the 2 or the 24, in terms of whether the computer would take it back away from you? or could take it back away?

F: Well, there's a possibility that he would take it away from me.

I: In either case?

F: Right.

I: Is one of these cases more likely than the other, do you think?

F: Well, it seems as if ... umm ... well the middle ones and the bottom ones ... so, ...

I: What about the middle ones and bottom ones?

F: Well, in the middle, that's where the computer mostly gets his, so ... and the bottom ones, he gets there also, so it's most likely in the bottom or the middle, so ...

I: So, if you had to choose between 2 and 24, which would you choose?

F: I'd choose 24.

A similar discussion took place at turn 8. Fred had rolled a four and a five and all three choices (1, 9, and 20) were unoccupied.

I: Which of the three are more likely and which are less likely to be taken away?

F: I think the less likely would be 20.

I: Why do you thnk that's so?

F: Because it isn't based on those that add up to 20.

I: What would you have to roll to get 20?

F: A four and a five. ...

I: Anything else?

F: Six and threee. ... No, that's 18. ... Uh, ... That's all.

I: Let's think about B. That's one; what would you have to roll to get one? (Interviewer gets paper and prepares to write.)

F: You could roll 5 - 4, 4 - 3, 3 - 2, 2 - 1, 6 - 5, and that's basically it.

I: What about one times one?

F: If you get two ones, if you're lucky enough.

I: What you can see from this list is that to get 20 you'd have to roll a four and a five, but to get one ...

F: You could roll (pointing to list) ...

I: All those different things, right?

F: Right.

I: What about the other choice, nine? Would nine be hard to get or easy to get, do you think?

F: Quite hard to get sometimes.

I: Well, what would you have to roll to get nine?

F: Three times three. ... seems to be it.

I: Could you add to get nine?

F: Yeah, eight plus one.

I: But there's no eight on the die.

F: Oh, yeah. Six plus three, five plus four, ... and that's it.

I: So what does all this tell you? (Shows the chart.) Is this information useful?

F: Oh, quite.

I: What use do you put this information to? What does it tell you to choose in this case?

F: Well, in this case I'd ... uh ... hope to get one of those and it added up to one they already had, so I could take theirs away.

I: So in choosing on this move, where you can't take any of them away, which of these do you choose?

F: The 20.

Again it should be noted that the exchange involved a great deal of prompting, so the questions of the depth of Fred's understanding and of the extent to which he could construct as well as follow such an argument remained unanswered. The interviewer was satisfied that Fred was indeed following the argument, but he doubted that Fred would have constructed it so elaborately on his own. It seemed likely that he would continue to rely on intuitions of probability rather than attempt to construct a logical argument.

Observations in Relation to the Major Questions of the Study

In Chapter 1 the problem of the study was specified in terms of seven major questions which were to be answered on the basis of experimental observation. An overview of the experiment with respect to the subjects individually has been presented in the previous section of this chapter. In this section the seven major questions are considered, and answers to them are offered on the basis of the experiment as a whole. These answers should be viewed in light of the limitations of the study, particularly its small and perhaps unrepresentative sample. For most of the questions it is not possible to answer with clear generalizations that are unequivocally supported by numerous observations. Nonetheless, relevant experiences from the experiment have been collected and summarized in the hope that insight, if not certainty, will be an outcome.

(1) What strategies do children spontaneously use in playing Capture?

Most people learning a new game begin playing without a strategy; they simply make some moves just to see what will happen. The children in this study were no different; most of them seemed to begin playing Capture 1 without a plan. One subject, Carl, began systematically, trying first A, then B, then C.

Many of the subjects considered several possible strategies in early moves of the game. Often these were based on and expressed in terms of relative positons of the markers of each of the players. For example, Alan considered "trying to close in on" Apple's markers, Dawn mentioned "going beside" them, and Fred thought of either "replacing" them or "going as far as possible away from them." In other trials of the games, children have thought of trying to get several of their men in a row. Students seemed to realize fairly quickly that there is no particular advantage to these strategies, and they soon abandoned them.

For most of the subjects, both strategies 1 and 2 should be considered spontaneously discovered strategies. It is usually possible to identify an experience in playing the game that resulted in the children noticing the value of these strategies, but most often the children appeared to be using these strategies before being questioned about them or being given relevant hints. When questioned they almost always gave clear and correct explanations for them. Most cases of non-use of these strategies can be traced to computational error, misreading of the number line, or failure to consider all possibilities and notice an opportunity.

It is fairly clear that Carl spontaneously discovered strategy 3, since he used and expressed the idea of choosing places that he thought the opponent would be less likely to recapture before being questioned along these lines. However, his assessment of the probabilities was not always accurate and consistent, so indicator variable S3 was not always positive even when he was using this strategy.

Elsa as well began using the idea of trying to choose places that would be less likely to be recaptured, and the interview record suggests she thought of this idea on her own. In the discussion of this strategy she showed insight and understanding, and her application of it was more consistent than any other subject's.

Beth recognized a need to choose moves in such a way that they would be less likely to be recaptured, but her attempts to implement this strategy failed, and it appears that she concluded that it was impossible to do so. Fred used strategy 3 to some extent, but it appears that his understanding of it resulted mainly from the leading questions he was asked about it.

Strategy 4 was used spontaneously, and for the most part accurately, by the two students who discovered strategy 3 on their own. Both they and other subjects spontaneously used a hierarchical analysis in choosing their moves which gave highest priority to capturing places held by the opponent. Other strategies were usually used only after the possibility of opponent capture was taken into account.

(2) Do children's strategies for playing Capture change as they gain experience playing? How?

An examination of children's spontaneous development of strategies indicates that strategies change as a result of experience, since children rarely, if ever, began playing Capture with completely formed views of how to win. Although they all read and claimed to understand the rules which included mention of capturing positions held by the opponent, very few children chose and recognized the value of an opponent capturing move before they observed Apple exhibiting this behavior. (Fred's behavior at the start of his first game is typical of this situation.) In this sense strategy 2 was developed as the result of the experience of having a position captured by the opponent. In some cases, notably the case of Beth, it was necessary to draw attention to changes in the board and in the score that result from an opponent capture.

Strategy 3 is also based on the experience of having one's positions recaptured by the opponent, and it is essentially an attempt to take defensive action to prevent this. One child, Carl, required only half a game's experience before recognizing this as a problem and attempting to solve it; others (e. g. Elsa and Fred) required more experience, while some children (e. g. Alan and Dawn) never took action even though they were more than once made aware of the situation by means of hints and pointed questions.

(3) Do children's strategies for playing Capture change as a result of adults' questions? How? As consequences of what specific questions?

It is always dangerous to infer a cause and effect relationship solely on the basis of evidence of correlation. In the present study there are several instances in which a question was asked by the interviewer and the child's behavior relevant to the question was subsequently different from previous behavior. Although it seems likely in these cases that the questions caused the changes in the children's thinking and behavior, there is no proof that this is the case. Perhaps a follow-up question such as "Did you change your thinking because of what I asked you?" (with reference to particular questions and/or particular thoughts) would be useful in supporting or refuting the case for causation, but there may be serious doubts about the validity of answers to such a question because it calls for a reflective judgement rather than a report of concurrent cognitive behavior. At any rate, it did not occur to the interviewer to use this tactic in the interviews, so the point is moot. These reservations notwithstanding, there does seem to be evidence that suggests that certain questions did affect certain children's thinking and behavior with respect to strategies.

The general question, "Does it matter which alternative you choose?" and its variants such as "Are some numbers better choices than others?" seem to have been effective in curtailing the period of playing without a plan just to see how the game goes that normally appears when players are learning a new game. In some cases this question seems to have precipitated a more thorough checking through each of the alternatives and, as a result, increased use of strategy 2. It may have suggested to some subjects that even when strategies 1 and 2 do not apply (as in the case when all the alternatives are free captures), there still might exist another strategy.

In the case of Beth the question "What happens to the score when you choose X (an opponent capture)?" seems to have caused the realization that opponent captures are more valuable than free captures, i. e. strategy 2. The difference in her behavior before and after this question (day 1, game 1, turn 9) is striking. (See Table A2.2.1.)

With students who did not seem to be understanding or using strategy 3, the following questions were used: "Have you noticed that some of the positions on the board seem to be captured by one player, then quickly recaptured by the opponent, and so on back and forth a number of times? Why is that?" This line of questioning had mixed results. With some children (e. g. Elsa) it seems to have played a role in increasing both the use and the verbalization of strategy 3, as was intended. With Alan it was not effective. Although he was able to recall several positons that had changed hands more than once during the preceding game, these were not uniformly the positions with the largest numbers of ways, and he did not seem able to predict that the places that would change hands most often would be those that can be captured with the greatest numbers of different dice rolls. When this same question was used again on the next day (day 3), his answer was much closer to the actual events. Although he seemed to improve in answering the objective question, he still did not have an explanation for the phenomenon or an appreciation of its implications for action.

Questions which prompted the children to list the dice rolls with which a player (in particular the opponent) could recapture each of the alternative positions in a given move also had limited success. In the first place, children usually had some difficulty producing these lists and deciding whether they were exhaustive. Second, even with complete lists, some of the children seemed reluctant to conclude that the lists proved anything. Others, for example Carl, Elsa, and Fred, seemed to recognize the connection between observed probability and predicted probability. When this question is used with the term 'chances,' as it was in the case of Elsa, the term itself may suggest a connection between the 'chances' of getting an outcome in the sense of the ways of getting it and the 'chances' of an outcome in the sense of quantification of randomness. But, in no case did a child have an "Ah ha!" reaction indicating that he recognized quantification of probability as a problem that could be solved by the counting procedure.

(4) How do children describe, justify, and explain their strategies?

Strategy 1 seemed obvious to most students, and this is probably the reason why some students seemed puzzled by the question, "What would happen if you chose to capture X (a position already held by the player)?" Most children replied simply, "I already have that one." Other children clearly and correctly explained that although a player could recapture one of the positions already held, doing so does not change the score. Dawn and Elsa both described it as a "waste of time," and Carl said it "uses a turn that could be used to gain one more."

Strategy 2 also seemed obvious to most children. When they were asked to explain why they thought it was better to capture an opponent's position than an unoccupied one, several children contrasted "I gain one and he loses one" with "I just gain one." None of the children referred to the fact that the relative score changes by two in the former case and by one in the latter, but several did tell correctly what the scores for each player would be in each case.

When Beth was still at the stage of not recognizing strategey 2, she described an opponent held position as "already used up." This expression is clearly related to her misconception about strategy 2.

The vocabulary used by the children in describing strategy 3 and their arguements justifying it are a main focus of the study. To be complete and correct, the description of strategy 3 should be in terms of a comparison of the probabilities of different positions being recaptured by the opponent later in the game. In the series of interviews with the six children none of them used the term 'probability,' although several referred to some events as being "more likely" than others or as having "better chances" of occuring. Carl repeatedly used the term 'possibility' for 'probability,' saying that one event had "less of a possibility" of happening than another. Fred labelled rolling doubles and a board position made using doubles as "lucky," and from the context it appears that he was referring to their rarity (i. e. small probability of occurring), for he said that while he was "lucky" to be able to choose that position, he doubted that Apple would be so lucky as to be able to take it away from him.

Many of the children did not use terms related to 'probabilities,' 'chances,' or 'likelihoods,' but instead framed their arguments in terms of predictions, and especially comparisons of predictions, of future events. For example, in explaining why they chose one place over another several children said that they thought that Apple would be able to recapture one position, but not another. The probabilistic nature of these predictions is indicated by the tentative tone that the children often used, and by their use of qualifiers such as 'probably,' as in one event "would probably happen," but another "would probably not happen." Elsa made a remark in terms of relative prediction, and then, as if to correct herself, made the same comment using the term 'is more likely.'

In general, these results suggest that the children had intuitions of probability, chance, and relative frequency, even though they did not have the technically correct vocabulary for expressing them. One of the difficulties involved in giving interpretations in this respect has to do with the fact that the interviewer deliberately attempted to avoid using probabilistic terms in asking children about their strategies, for fear of introducing a pattern of thinking that might not have been used spontaneously by the children. Instead, his questions sometimes suggested predictions, and this may have contributed to the children's extensive use of prediction and comparison of predictions as noted above. In a few cases he framed his questions in terms of some events being 'more likely' than others, and several instances of the children using this expression were also noted. In one case where the interviewer used the term 'chances' before it was used by the interviewee, the child, Elsa, subsequently (and perhaps as a result) used this term repeatedly in describing and justifying her strategy. As noted above, this term is interesting because it can suggest both theoretical probability where the outcomes being counted are considered 'chances,' as well as empirical probability when children are asked to estimate the 'chances' of some familiar event.

Those students who described and used strategy 3 usually but not always coordinated its use with strategies 1 and 2, in effect using a version of strategy 4. In describing this, students most often reported that they worked out the values for each of the alternatives, checking to see whether any of them permitted an opponent capture and ruling out any that were self-captures. Strategy 3 might then be applied if there were more than one choice in the most desirable category. Some students used strategy 3 only when choosing among free captures, i. e. they used strategies 1 and 2 or 3.

(5) Are children's moves in playing Capture consistent with the strategies they claim to be using to determine them?

In the interviews many instances were found of disagreement between the strategies children claimed to be using and the variables S1, S2, S3, and S4 whose values reflect the use and non-use of these strategies. This fact underscores the need to consider both the quantitative data collected by the computer and the qualitative data obtained from the children's verbalizations. If the analysis were restricted to the former only, the view of the extent of the children's understanding and ability to apply would certainly be more negative than it is when both sources of information are taken into account. On the other hand, it is important to recognize that the children did not always act on what they claimed to understand. In many cases reasons for the discrepancies between the children's overt behavior and the strategies they claimed to be using can be found quite easily, as the following examples show.

Elsa had discovered strategies 1 and 2, yet on several consecutive moves she recaptured a position she already held instead of one held by Apple, and her dismay at these errors was becoming increasingly obvious. After several moves she overcame this difficulty; only then did she explain what her problem had been and how she had solved it. Her error had been to mistake the 0 position on the number line, which is formed by a letter I on the Apple text screen, for 1. As a result, each time she attempted to capture position 1 which was held by Apple, she captured 2 which she already held.

Carl rolled multiplication several times in succession in Capture 2 while Apple held 0, yet he did not choose to recapture 0 by using 0 as the player supplied factor. When asked why he did not recapture 0, he replied that he had forgotten about that possibility.

In a few cases in Capture 1 the children seemed surprised that the letter they chose had the result it did on the number line display; apparently a computational error led to the capture of a position different from the one they intended. Alan, who was the weakest in basic facts, had this experience more often than the other children.

In many cases an apparent failure to use strategy 3 can be traced not to a failure to understand the strategy, but to an inability to determine correctly which alternative theoretically has the smallest probability of being recaptured. Carl, for example, assumed, apparently on the basis of actually observed frequencies, that 0 has a lower probability of recapture than 33 in Capture 1. On the basis of theoretical probability the opposite is the case. The limited success that resulted from questions which prompted children to compare lists of the dice rolls permitting various positions to be captured, suggests that these children may be capable only of a version of strategy 3 that is based on intuitions of probabilities of recapture. Unfortunately these intuitions are not always correct since they seem to stem from the most readily recalled ways of making the positions and not from formally compiled lists of all the ways. Thus variables S3 and S4 may take on the value -1 indicating that the child's behavior is inconsistent with strategies 3 and 4 when, in fact, these strategies are understood and are being applied, but the needed probability distribution has not been calculated or estimated correctly.

(6) To what extent do children use mathematical concepts in determining and explaining their strategies for playing Capture?

As has been indicated above, especially with reference to questions 4 and 5, the children used strategies 1 and 2 more readily and consistently and described and justified them more clearly and confidently than was the case with strategies 3 and 4. This result is not surprising since the mathematical concepts on which strategies 1 and 2 are based are notions of addition and subtraction as applied to changes in each player's score and to comparisons between the player's score and the opponent's. Children in grades 4 to 6 are normally expected to be thoroughly familiar with these concepts and able to apply them to situations such as these, and these expectations were confirmed by the experiment.

The situation with respect to the probability concepts on which strategies 3 and 4 are based is quite different. In the first place, instruction in probability per se was not part of these children's school experience. Although the children usesd the terms 'probably,' 'likely,' and 'chance' in various contexts, they never used the word 'probability.' Secondly, the review of relevant research suggested that even in the absence of instruction children of this age have usually begun to develop some concepts related to probability, but that this development is by no means complete with respect to the cognitive operations and intuitions involved.

It is probably more accurate to characterize the ideas which form the basis for these children's strategies as intuitions rather than concepts. Those students who exhibited understanding of strategy 3 used comparisons based on intuitive predictions of future events or intuitive estimates of their probability as the basis for choosing their moves. In some cases it seemed clear that it was observed probability rather than theoretical probability notions that were involved.

Carl, Dawn, Elsa, and Fred all stated and used the generalization that larger numbers are less likely to be recaptured than smaller ones. While this observation tends to be correct, the distributions of theoretical probabilities actually have shapes like the shapes of the graphs in Figure 3.2, and there are examples in which for a given dice roll the alternative with the largest value does not have the smallest probability. For example, with the roll of a three and a six in Capture 1, 18 has a lower probability than 36, and for four and times in Capture 2, both 28 and 32 have lower probability than 36. None of the children seem to have considered the possibility of such exceptions to the generalization.

None of the children spontaneously produced a detailed argument using concepts of counting to justify strategy 3. Carl and Elsa seemed to be arguing implicitly that the high numbers which could be made only with place value alternatives had fewest ways. When asked about the ways for 0 in Capture 1, Carl readily observed that there would be six, corresponding to the six doubles. Some of the children who were asked to make lists of the dice roll outcomes that permitted the capture of various alternatives seemed to be able to understand and apply the facts they were led to discover, but they had difficulty producing the lists and determining whether or not they were complete.

(7) To what extent do children generalize or transfer their strategies for playing Capture and the mathematical concepts on which their strategies are based from one version of Capture to another?

Capture 1 and Capture 2 are really quite similar, and the impression of similarity is reinforced by their similar names and their similar appearances on the computer's video screen. Some children moved quite quickly when reading the rules for Capture 2. This was particularly true of Fred, who had no ready answer when he was asked, after he had read the rules, how Capture 1 and Capture 2 were different. Other children commented either "that's the same" or "that's different" at the presentation of each frame of the rules for Capture 2 and seemed to have a good grasp of the similarities and differences.

In order to gain insight into this major question, the children were asked, "Did you learn anything from playing Capture 1 yesterday and the day before that helped you in knowing how to play Capture 2?" Surprisingly, some of the children answered this question in the negative. Most of the children, however, said they found it easier to learn to play Capture 2 because of the experience of playing Capture 1. When asked about similarities and differences between the games, several children remarked that they were "just the same," and went on to tell how they used the ideas of strategies 1 and 2 in both games. Those children who had developed strategy 3 for Capture 1 also used it in Capture 2, and they cited the generalization about larger numbers being less likely to be recaptured than smaller ones as if it applied to Capture 2 as well.

Most of the children thought that Capture 2 was a harder game than Capture 1, and they cited the fact that a player has more choices in Capture 2 and the fact that the alternatives are not specified on the screen as their reasons. Alan said he thought Capture 2 was easier, but he gave no clear reason for this opinion.

CHAPTER 5

DISCUSSION

In this final chapter, the investigation as a whole is discussed, and reflections of four main types are presented. First, the study is summarized in order to give an integrated view of the four preceeding chapters. Second, conclusions are drawn. Third and fourth, implications of the study for future research and for educational practice are considered.

Summary

This study is relevant to the current state of affairs in elementary mathematics education because it relates to three areas of concern: (1) the use of games in teaching, (2) the introduction into the curriculum of topics in probability, and (3) the selection and development of effective and worthwhile microcomputer based activities for elementary school children. A teaching experiment in the experiencing mode was conducted to investigate children's thinking about probability as it develops from and is applied to the playing of two versions of a microcomputer game. The teaching method being studied involved both experience playing the games and questions about the games and the children's thinking as they played. Task based interviews and thinking aloud procedures were used; qualitative data gathered from the interviews were coordinated with and compared to quantitative data (gathered by routines in the computer programs) on the extent to which the children's play was consistent with known strategies for playing the games.

Research related to children's development of notions of probability suggests that children normally develop certain understandings and abilities related to probability that are relevant to playing and learning from games such as Capture. These understandings and abilities are connected with the development of intuitions and cognitive operations, and this development takes place over time even in the absence of formal instruction. The children involved in this study, who were in grades four, five, and six and who ranged in age from about 10 to 12 years, exhibited various levels of understanding and ability to apply probability ideas, as would be predicted on the basis this research. Although none of the children consistently used technically correct probability terminology, as would be expected since the children had not been given prior formal instruction in probability, qualitative assessment of their statements about their play indicated that some of them did have understanding of and ability to apply probability ideas based on intuitions. Some children applied probability concepts correctly even before being asked questions that might have suggested to them that they should do so. Others recognized that some board positions changed hands more frequently than others, but did not seem able to relate this information to estimates of the probability of recapture.

As would be predicted on the basis of research on children's abilities with combinatorial operations, the children in this study had difficulty producing organized and exhaustive lists of the ways in which positions could be recaptured. As a result, they made some errors in implementing the strategies they claimed to be using. Other discrepancies between the children's statements about their strategies and their actual play seem to have resulted from computational errors or other oversights.

The role of the computer in this study was twofold. On the one hand it interacted with the students, presenting the rules, displaying the moves of the game, and acting as one of the players. On the other hand it served a record keeping function, collecting detailed data on the course of each game and analyzing each play in terms of four variables which reflect the use or non-use of identified strategies. The study demonstrated that it is feasible for the computer to serve each of these two functions. The success of the computer system in the first role has implications for future educational practice; implications for the methodology of future research studies can be drawn from the results of using the computer in the second role.

Conclusions

The findings of this investigation do not lead directly to conclusions. The main reason for this is that the goal of the study was not to draw conclusions of the form 'product A is superior to product B,' or 'trait C is associated with trait D,' or 'treatment E produces outcome F.' Instead the goal has been to explore an educational process and attempt to understand children's thinking as it develops in and is applied to that situation. However, in the process of asking seven specific questions and answering them on the basis of experimental evidence, many observations have been made which lead to and support the following three main conclusions.

(1) Playing the games Capture 1 and Capture 2 is a suitable activity for elementary school children in grades four to six.

The children who took part in this study found playing the games enjoyable and interesting; one child added, "It makes you think." There were no technical difficulties with the games. The level of ability required in reading the rules and directions, interpreting the graphic displays, and performing needed arithmetic computations seemed appropriate. In each move of the game the children selected and performed several computations and used other cognitive skills such as planning ahead, predicting outcomes, and evaluating alternatives. Some children applied intuitions of probability in the construction of strategies for playing, even without being given questions or hints. Failure to recognize a strategy did not seem to affect children's interest or willingness to play.

(2) Discussion accompanying the games is especially worthwhile.

There is evidence to suggest that certain questions asked by the interviewer increased children's use of and ability to verbalize strategies. Although children tended not to use technically correct vocabulary in expressing probabilistic concepts, many of them did apply intuitions of probability in constructing strategies. Discussion of strategies can provide the children with contexts in which to use probabilistic terminology correctly and settings in which to apply mathematics concepts.

The method of playing and discussing games that has been investigated in this study puts the teacher and the computer into distinctive roles. The computer structures learning situations for children in which there are natural opportunities to apply computational skills as well as higher order cognitive abilities. When children work together using a computer, discussion often results spontaneously, and the use of a computer simulated player whose play models certain specific strategic behaviors can spur the discussion as well. The teacher's function is to monitor the children's play and to enhance their thinking and discussion by asking questions about the games and suggesting features of the games that should be heeded. These roles for the computer and the teacher in the computer assisted classroom are quite different from the roles adopted when drill and practice software is used, when computer tutorials are presented, when the computer is used for record keeping or report generation, or when children learn programming languages such as BASIC or Logo. The use of Capture in this style has been shown to be profitable, and it seems likely that other games have potential to be used in similar ways.

(3) The methodology used to investigate the questions of the study was appropriate and fruitful.

The analyses of results showed that the children in the study differed considerably with respect to their use of strategies and their ability to explain and justify them. This demonstrates that the methodology was sensitive enough to detect differences among children that are of interest. It was observed in many cases that both quantitative and qualitative data were needed in order to interpret children's thinking correctly. For example, some children understood and could explain strategy 3, but when they attempted to use it, they made moves inconsistent with it because they estimated the probabilities of recapture incorrectly. On the other hand, even random behavior is sometimes consistent with an identified strategy, so the children's verbal reports of their thinking are needed in order to characterize the thinking on which the overt behavior is based. By combining and coordinating the two types of data analysis, useful insights into the children's thinking were obtained.

Although qualtative research and research of an exploratory nature have many notable limitations, valuable insights can still be gained. The results of this study provide a basis for future investigations; implications for further research are considered in the following section.

Implications for Further Research

This study has implications for future research which seem to fall into two main categories. First are implications for the content of future research efforts; in this section possible replications and extensions and variations of the present study are considered. Some of these arise from the acknowledged limitations of the present study, while others are based on an assessment of opportunities to extend the present study and its methods to other games and the mathematics topics they entail and to other problem solving situations.

The second set of implications is drawn from reflections on the methodology used in the study. Since it has been concluded that the methodology used was appropriate and fruitful, implications for its future use are considered. Observations on the coordination of quantitative data collection and analysis procedures and qualitative data collection and analysis procedures are given, and recommendations are made regarding present and possible future technologies for these operations.

Implications for the Content of Further Research

Several lines along which the research begun in this study might be extended are suggested by the list of limitations that was presented in Chapter 1. The first limitation mentioned was that this study was not designed to compare its student activities and their effects with other, competing activities and their effects. Following on the conclusion that the games and certain questions about them are worthwhile activities for children to be engaged in, a logical next question would be whether these activities are any more or less worthwhile than other specified activities that have also been judged to be worthwhile. To investigate this question a ´teaching experiment in the testing mode´ might be in order, but such a project would demand careful selection of the competing activities, clear definition of the meaning of the terms ´worthwhile´ and ´more worthwhile,´ and attention to the specific educational goals shared by the sets of activities being compared. Although such an investigation seems attractive because of the definiteness that could be expected from its conclusions, it, too, would have its limitations, and the control of intervening variables and the construction of agreed upon measures of ´worthwhileness´ would probably present major difficulties.

The second limitation of the present study noted in Chapter 1 was that situations involving interactions between children playing the games together were not included. This limitation was imposed for the sake of simplicity and convenience, and not becuase situations in which children interact with their peers are considered any less important than situations in which students play individually against a computer simulated opponent. In fact, the opposite may be the case; recent activities outside of the present investigation have shown that having two or more students play as a team against Apple and against each other often result in rich verbal and non-verbal interactions. Moreover, this organizational pattern is actually used more often in schools, where access to microcomputers tends to be restricted. For these two reasons, an extension of the study to include peer interaction would be desirable.

The third limitation was that the use of specific interviewer questions in specific orders and in response to specific situations was not systematically investigated. In the interviews that were conducted, a variety of questions were asked, but their timing and selection were left to the judgement of the interviewer as he assessed the interview situations. In some cases it is clear that his questions were opportune, because, for example, the children sometimes finished his sentences for him and began to answer his questions before they were completely spoken. At other times the children´s responses seemed to indicate that they had not been thinking along the lines that the interviewer had anticipated, and the questioning seemed to interrupt the flow of their thinking and verbalization. These facts suggest that that qualitatively important interactions (some beneficial, some not) occurred between the interviewer and the subjects. Replication would be warranted so that the dynamics of interviews conducted by other persons who might assess interview situations differently and choose different questions could be compared with the interviews reported in this study.

Although the effects of the use of specific questions in specific situations could not be investigated systematically and completely without a much larger sample and more careful advance planning of the interviews, the findings of the present study suggest that replication would be warranted so that certain hypotheses about interviewer questions could be investigated in a limited way. One hypothesis is that interviewer questions involving prediction tend to cause children to think and to verbalize their thinking in terms of prediction or comparison of predictions rather than in terms of probabilities, likelihoods, or chances of outcomes. A second hypothesis is that interviewer use of the term "chances" tends to cause children to use this term more frequently in their subsequent verbalizations and that it suggests to them a connection between theoretical probability ideas and experimental probability ideas.

The fourth limitation mentioned in Chapter 1 was that the study was limited to a small number of subjects in a particular school. Since the sample actually used cannot be considered to be representative of the population of all students in grades four to six, replication involving other subjects might be valuable. However, the reservations expressed in Chapter 3 with respect to the ratio of costs to benefits in such cases still apply.

This study has focussed on particular games, Capture 1 and Capture 2, and specific mathematical concepts, namely probability concepts, that may be developed and applied as children play and discuss the games. At the present time the array of educational games is vast and rapidly growing. Since many of these games involve applications of mathematical concepts and skills in a similar way, it is easy to imagine studies parallel to this one involving other games and the mathematical concepts and skills they demand and the discussion questions teachers would employ in helping children develop strategies for them. Examples of games which might be investigated include ´Ladder´ (a game involving ordering of two-digit numbers and prediction of outcomes with a rectangular probability distribution), many of the common number guessing games (such as variants of ´Hi-Lo´ and ´Trap´), and perhaps some of the ´strategy´ games and ´computational/strategy´ games used by Goldberg (1980) and Fluck (1982) in their studies of the impact of game playing on children´s problem solving achievement. The fact that a number of mathematics educators are now working to develop strategy games for use as vehicles for mathematical problem solving instruction suggests that many games appropriate for such investigations could be found. This sort of research would be relevant to research in the areas of the nature of problem solving, the cognitive characteristics of problem solvers, and problem solving instruction.

Implications for the Methodology of Further Research

A central feature of the research methodology used in this study is its reliance on both quantitative and qualitative data. As Chapters 3 and 4 show, two fundamentally different kinds of information are collected using distinct sets of apparatus, but the analysis and interpretation of the data requires their coordination and integration. In the present study the interface between the two data collection and analysis systems was a human one, the investigator himself. A great deal of time was spent making accurate transcriptions of the audio tapes and matching events in the transcripts with the computer generated records of the games. An estimate of the time required to produce a transcript would be about three to five times the length of the actual interview session. Searching through the transcripts, editing them, and identifying and categorizing excerpts to be included in the report takes additional time.

The most bothersome part of this process was the construction of the correspondence between events on the audio tape and turns in the games. On some turns the players took much more time than on others, and there were sometimes periods of several turns when nothing was said. In retrospect, several things could have been done to make this process easier. Greater use of the tactic of verbally identifying the turn of the game when asking questions about it, which the interviewer used only occasionally, would have helped. Also, routines could have been added to the computer programs to generate coded markers (e. g. audible tones) to indicate the progress of the game. A relatively simple implementation of this idea would be to have the Apple generate beeps of different pitches following players' key-presses when prompting Apple's move, when choosing their own move, when going on the the next turn, and so on. Audible markers such as these would have been much easier to interpret than the ordinary noises emanating from the keyboard, although even these noises sometimes gave useful clues about what was happening on the tapes. A more sophisticated implementation of the same basic idea, and one that would be less distracting, would be to program the computer to generate markers and send them to the casette tape output port. A stereo tape recorder could then be used to record the voices of the child and the interviewer on one channel and the coded information about the turn, player, dice roll, and chosen alternative on the other channel. Alternatively, it might be possible to use the computer's video output as video input to a video tape recorder receiving audio input from a microphone. In the future other hybrid technologies will surely become available.

The computer programs that read the records of the games and produce printed records and analyses in terms of values for the four variables S1, S2, S3, and S4 worked quite efficiently. Their features could be incorporated into the main programs so that records being made would include analysis as well as raw data. Although it is possible to calculate the values of variables such as these by hand, doing so is extremely time consuming, and it is deceptively easy to make errors in the process. For instance, when debugging the computer routines that calculate S1, S2, S3, and S4, the investigator/programmer found that discrepancies were often due not to bugs in the routines but to human logical errors. The routines and programs that evaluate these four variables at each move of the game were extremely valuable in documenting assertions that children's thinking included specific strategies and resulted in particular overt behaviors, and it is fortunate that information like that presented in Appendix 2 was so readily available. In planning future studies, careful consideration should be given to the behaviors that are expected to result from anticipated patterns of strategic thinking, so that appropriate behavioral indicators can be constructed and later processed by computer. Such variables might be dichotomous, integer-valued, or real-valued, but their purpose should be to quantify the extent to which specific kinds of thinking are reflected in action.

Implications for Practice

Several months before the present investigation was begun, before Capture 2 had been invented, and before the routines needed for the games to be played against a computer simulated opponent had been developed, the author wrote an article for the Arithmetic Teacher (Schroeder, in press) describing Capture 1 for two human players and outlining some suggestions for elementary school teachers. Recent developments have strengthened that article's rationale and suggested additional points that can now be included in some suggestions for practice that are given in the following paragraphs.

Capture 1 and Capture 2 for two human players or one human and one computer simulated player are suitable activities for elementary school children, and experience suggests that children can learn to play the games with minimal, if any, teacher intervention. Experience also suggests that when two or more children play the games together, they often discuss the game, the score, and their strategies, and make predictions about the outcomes of various courses of action. This sort of rich verbal interaction (as well as the non-verbal communication that often accompanies it) should be encouraged by the teacher. Without some sort of action on the part of the teacher, children may not realize all the possible benefits that can be derived from playing and thinking about and talking about the games.

Questions, both specific to situations in the games and general ones, can be used to help children think reflectively, become aware of and express their intuitions, analyze situations in terms of mathematics concepts, and apply mathematics. Table 3.7 suggests some appropriate questions and indicates circumstances under which they may be especially relevant; additional potentially useful questions are mentioned in Chapter 4 and in the article. The cycle of playing, discussing the games and strategies for playing them, playing again, and so on can be repeated several times with appropriately chosen questions each time.

Certain activities initiated and directed by the teacher may be useful in developing the children's intuitive and operational knowledge of probability. This goal can be approached through questions which lead to strategy 3. A likely starting point would be discussion about the fact that moves in which the opponent recaptures a position held by the player are undesirable, leading up to the question "Can anything be done about this?" Either the children or the teacher may reinterpret this question as "Are some positions more likely than others to be recaptured by the opponent later in the game?" Experience suggests that some children may have an intuition that "high numbers" tend to have the lowest probabilities of being recaptured, but other children may not have any feeling at all for an answer to this question.

Since for children of this age observed probability notions which depend upon intuitions of relative frequencies in concrete situations seem to be more compatible with their level of cognitive development than theoretical probability notions which rely upon combinatorial operations characteristic of the formal operational stage, the first steps of guided discovery should be based on an actual experiment. Collecting data and constructing a graph to show the number of times each of the positions 0 to 36 was able to be recaptured in a large number of turns would be a relevant activity at this point, and one that would be appropriate in view of the comprehensive mathematics curriculum of most North American elementary schools. Discussion based on this graph could include whether the experiment supports the generalization that the larger numbers tend to be less likely to be able to be recaptured, and other questions.

A second series of activities could deal with strategy 3 from the point of view of theoretical probability. One step would be to pose the following problem: for each board position 0 to 36, which dice roll outcomes permit a player to capture that place? Since it was found that the children in this study had some difficulty answering this question for small numbers of selected positions, it might be prudent to spend some time discussing potential problem solving strategies for use with this problem or to arrange for several small groups of students to work on the problem and afterwards compare their solutions and their methods. Incidentally, it might be tactically advantageous to do this investigation first with Capture 2 rather than Capture 1, because there are fewer dice roll outcomes (18 as opposed to 36), and because the issue of whether rolling an X and a Y is the same as rolling a Y and an X does not arise. From the lists of the dice rolls that permit the capture of each position can be constructed graphs showing the number of ways for each position (i. e. the graphs shown in Figure 3.2). Discussion of these graphs for Capture 1 and Capture 2 can be generated by a number of questions, including, for example, reasons for similarities and differences between these graphs and the corresponding ones constructed from the probability experiments.

Comparisons between Capture 1 and Capture 2 can be clarified by comparing the tables and graphs for each game that have been described above. Experience suggests that children tend to overgeneralize because of the similarities between the games, so questions about both similarities and differences are needed.

These suggestions for using Capture 1 and Capture 2 involve children in some activities where they interact with a computer and other activities where they reflect on those activities under the guidance of their teacher. The objective of these activities was the development of understanding and application of probability and of counting concepts; other games involving other mathematical ideas could also be introduced into classrooms using similar methods. For example, the connections between estimation skills and various number guessing games, or the connections between Nim and problem solving heuristics such as working backward and searching for patterns, or the connections between Bagels (a variant of Mastermind) and logical deduction, systematic trial and error, and other heuristics might be exploited. While learning activities with these games and mathematical ideas would probably involve different discussion questions, the general method of using play and discussion would be quite similar.

The suggestions outlined above are just that, suggestions. They stem from the belief that in the hands of a good teacher, one with a mathematical cast of mind and a penchant for problem solving, they can become rich experiences that children find motivating. They are offered in the hope that through them both teachers and children will have opportunities to enjoy investigating the world and solving some of its problems with mathematics.

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APPENDIX 1

FORMS USED TO OBTAIN CONSENT

The following forms were used to obtain the informed consent of the subjects' parents:

(1) Letter to Parents

(2) Research Permission Form

THE
UNIVERSITY
OF CALGARY

Faculty of EDUCATION
Department of
EDUCATIONAL CURRICULUM AND INSTRUCTION

2500 University Drive N.W., Calgary, Alberta, Canada T2N 1N4

Telephone (403) 284-5639

Dear Parents,

For several years I have been involved in teaching university courses for teachers on the subject of teaching mathematics in elementary schools. I have also been involved in developing mathematics activities and materials for use with children. Lately I have been working on microcomputer programs to be used in elementary school mathematics classes.

I am now in the process of carrying out research to evaluate the effects of new educational games designed to help children develop and apply mathematics concepts. As children play these games they not only practice arithmetic operations, but they also have to decide what moves to make. How they make such decisions on the basis of their understanding of mathematics is of particular interest in this research.

If you give your permission, I would like to interview your child while he or she plays two educational games I have designed. The purpose of the interviews is to find out how children react to the games and to some questions about the games. Individual interviews lasting about 20 to 30 minutes each will take place on three or four days during school time. The staff at University Elementary School support this activity as a worthwhile educational experience.

If you give your permission for your child to take part in this activity, please fill out and return the form attached. If you have any questions about this research or about your child's participation in it, please do not hesitate to call me at my office, 284-6173 or 284-5645.

I sincerely thank you for your cooperation.

Yours truly,

Thomas L. Schroeder
Assistant Professor of
Curriculum and Instruction

Enclosure

RESEARCH PERMISSION FORM

Child's Name ....

Date of Birth ....

Parent's Name ....

Address ....

Telephone ....

I give permission for my child to participate in individual interviews while playing educational games designed by T. L. Schroeder. I understand that the purpose of the interviews is to gather information about the effects of using these games to help children develop and apply mathematics concepts. I also understand that recordings of my child's responses during the interviews may be made, but that these recordings will be destroyed at the conclusion of the research project and not made public in any form in which my child could be identified.

Parent's Signature ....

Date ....

APPENDIX 2

QUANTITATIVE DATA SUMMARY

This Appendix consists of the following tables:

Table A2.1.1: Quantitative Data for Alan, Day 1
Table A2.1.2: Quantitative Data for Alan, Day 2
Table A2.1.3: Quantitative Data for Alan, Day 3
Table A2.2.1: Quantitative Data for Beth, Day 1
Table A2.2.2: Quantitative Data for Beth, Day 2
Table A2.2.3: Quantitative Data for Beth, Day 3
Table A2.3.1: Quantitative Data for Carl, Day 1
Table A2.3.2: Quantitative Data for Carl, Day 2
Table A2.3.3: Quantitative Data for Carl, Day 3
Table A2.4.1: Quantitative Data for Dawn, Day 1
Table A2.4.2: Quantitative Data for Dawn, Day 2
Table A2.4.3: Quantitative Data for Dawn, Day 3
Table A2.5.1: Quantitative Data for Elsa, Day 1
Table A2.5.2: Quantitative Data for Elsa, Day 2
Table A2.5.3: Quantitative Data for Elsa, Day 3
Table A2.6.1: Quantitative Data for Fred, Day 1
Table A2.6.2: Quantitative Data for Fred, Day 2
Table A2.6.3: Quantitative Data for Fred, Day 3

Table A2.1.1: Quantitative Data for Alan, Day 1

	Turn	S1	S2	S3	S4
Game 1	1	0	0	1	1
	2	1	0	1	1
	3	0	0	-1	-1
	4	0	0	-1	-1
	5	1	0	-1	-1
	6	0	-1	-1	-1
	7	0	1	1	1
	8	0	0	-1	-1
	9	1	1	1	1
	10	1	1	0	1
	11	0	1	1	1
	12	0	0	1	1
	13	0	-1	-1	-1
	14	1	-1	1	-1
	15	1	-1	1	-1
	N(1)	6	4	8	7
	N(0)	9	7	1	0
	N(-1)	0	4	6	8
	Turn	S1	S2	S3	S4
Game 2	1	0	0	-1	-1
	2	0	0	-1	-1
	3	0	1	0	1
	4	0	1	0	1
	5	-1	0	-1	-1
	6	1	1	0	1
	7	0	1	0	1
	8	1	1	0	1
	9	0	0	-1	-1
	10	1	0	1	1
	11	0	1	1	1
	12	0	1	-1	-1
	13	1	1	0	1
	14	1	1	0	1
	15	1	1	0	1
	N(1)	6	10	2	10
	N(0)	8	5	8	0
	N(-1)	1	0	5	5

Table A2.1.2: Quantitative Data for Alan, Day 2

	Turn	S1	S2	S3	S4
Game 1	1	0	0	-1	-1
	2	0	0	-1	-1
	3	0	0	1	1
	4	0	0	-1	-1
	5	0	1	0	1
	6	0	1	0	1
	7	0	1	0	1
	8	1	0	-1	-1
	9	0	1	0	1
	10	1	0	1	1
	11	0	-1	1	-1
	12	1	1	-1	-1
	13	1	1	0	1
	14	0	1	1	1
	15	0	1	1	1
	N(1)	4	8	5	9
	N(0)	11	6	5	0
	N(-1)	0	1	5	6
	Turn	S1	S2	S3	S4
Game 2	1	0	0	-1	-1
	2	0	0	-1	-1
	3	0	1	0	1
	4	0	0	-1	-1
	5	0	1	0	1
	6	0	1	0	1
	7	1	0	1	1
	8	0	0	-1	-1
	9	1	1	0	1
	10	1	1	0	1
	11	1	0	-1	-1
	12	1	1	1	1
	13	1	0	1	1
	14	0	1	0	1
	15	0	1	1	1
	N(1)	6	8	4	10
	N(0)	9	7	6	0
	N(-1)	0	0	5	5

Table A2.1.3: Quantitative Data for Alan, Day 3

	Turn	S1	S2	S3	S4
Game 1	1	0	0	1	1
	2	0	1	1	1
	3	1	0	-1	-1
	4	1	1	1	1
	5	1	1	0	1
	6	1	0	1	1
	7	1	0	-1	-1
	8	1	1	1	1
	9	1	-1	-1	-1
	10	1	1	0	1
	11	1	1	1	1
	12	1	0	-1	-1
	13	1	1	1	1
	14	1	0	-1	-1
	15	1	-1	1	-1
	N(1)	13	7	8	9
	N(0)	2	6	2	0
	N(-1)	0	2	5	6
Game 2	Turn	S1	S2	S3	S4
	1	0	0	-1	-1
	2	0	1	1	1
	3	0	-1	-1	-1
	4	1	1	1	1
	5	1	0	-1	-1
	6	1	1	0	1
	7	0	1	1	1
	8	1	1	0	1
	9	1	1	0	1
	10	1	-1	-1	-1
	11	1	1	-1	-1
	12	1	1	0	1
	13	1	1	-1	-1
	14	1	1	1	1
	15	1	1	0	1
	N(1)	11	11	4	9
	N(0)	4	2	5	0
	N(-1)	0	2	6	6

Table A2.2.1: Quantitative Data for Beth, Day 1

	Turn	S1	S2	S3	S4
Game 1	1	0	1	0	1
	2	0	1	0	1
	3	0	-1	-1	-1
	4	1	0	-1	-1
	5	0	0	-1	-1
	6	0	-1	-1	-1
	7	0	0	-1	-1
	8	-1	-1	-1	-1
	9	0	1	-1	-1
	10	0	1	-1	-1
	11	1	1	1	1
	12	1	1	1	1
	13	1	0	1	1
	14	1	1	0	1
	15	1	0	1	1
	N(1)	6	7	4	7
	N(0)	8	5	3	0
	N(-1)	1	3	8	8

	Turn	S1	S2	S3	S4
Game 2	1	0	0	-1	-1
	2	0	1	0	1
	3	0	1	0	1
	4	1	0	1	1
	5	0	0	1	1
	6	1	0	1	1
	7	0	1	0	1
	8	0	0	-1	-1
	9	0	1	0	1
	10	1	1	0	1
	11	1	0	-1	-1
	12	0	1	0	1
	13	0	1	-1	-1
	14	0	1	-1	-1
	15	1	1	0	1
	N(1)	5	9	3	10
	N(0)	10	6	7	0
	N(-1)	0	0	5	5

Table A2.2.2: Quantitative Data for Beth, Day 2

	Turn	S1	S2	S3	S4
Game 1	1	0	0	-1	-1
	2	0	0	-1	-1
	3	0	0	-1	-1
	4	1	1	0	1
	5	0	1	0	1
	6	1	1	0	1
	7	0	1	0	1
	8	1	1	0	1
	9	0	1	0	1
	10	0	1	0	1
	11	1	0	-1	-1
	12	1	1	-1	-1
	13	1	0	-1	-1
	14	0	1	1	1
	15	0	1	0	1
	N(1)	6	10	1	9
	N(0)	9	5	8	0
	N(-1)	0	0	6	6
	Turn	S1	S2	S3	S4
Game 2	1	0	0	-1	-1
	2	0	0	-1	-1
	3	0	0	-1	-1
	4	0	1	0	1
	5	0	0	-1	-1
	6	0	1	1	1
	7	1	1	0	1
	8	0	1	0	1
	9	1	1	0	1
	10	0	1	-1	-1
	11	0	-1	-1	-1
	12	1	-1	1	-1
	13	0	1	-1	-1
	14	1	1	0	1
	15	0	1	-1	-1
	N(1)	4	9	2	6
	N(0)	11	4	5	0
	N(-1)	0	2	8	9

Table A2.2.3: Quantitative Data for Beth, Day 3

	Turn	S1	S2	S3	S4
Game 1	1	0	0	-1	-1
	2	0	0	-1	-1
	3	0	0	-1	-1
	4	0	1	-1	-1
	5	1	0	1	1
	6	0	1	-1	-1
	7	1	-1	-1	-1
	8	0	1	-1	-1
	9	-1	0	-1	-1
	10	1	0	1	1
	11	1	1	-1	-1
	12	1	1	1	1
	13	1	1	1	1
	14	-1	-1	-1	-1
	15	1	1	1	1
	N(1)	7	7	5	5
	N(0)	6	6	0	0
	N(-1)	2	2	10	10
	Turn	S1	S2	S3	S4
Game 2	1	0	1	1	1
	2	0	0	-1	-1
	3	0	0	-1	-1
	4	0	0	-1	-1
	5	1	0	-1	-1
	6	1	1	1	1
	7	1	-1	-1	-1
	8	1	1	-1	-1
	9	1	1	-1	-1
	10	0	1	1	1
	11	1	-1	-1	-1
	12	1	-1	-1	-1
	13	1	-1	-1	-1
	14	1	1	1	1
	15	0	1	1	1
	N(1)	9	7	5	5
	N(0)	6	4	0	0
	N(-1)	0	4	10	10

Table A2.3.1: Quantitative Data for Carl, Day 1

	Turn	S1	S2	S3	S4
Game 1	1	0	0	-1	-1
	2	1	0	-1	-1
	3	1	0	-1	-1
	4	0	0	-1	-1
	5	1	0	-1	-1
	6	0	1	-1	-1
	7	0	1	-1	-1
	8	0	0	1	1
	9	0	1	1	1
	10	1	1	0	1
	11	0	1	-1	-1
	12	1	0	1	1
	13	0	1	-1	-1
	14	1	1	0	1
	15	1	0	-1	-1
	N(1)	7	7	3	5
	N(0)	8	8	2	0
	N(-1)	0	0	10	10
Game 2	Turn	S1	S2	S3	S4
	1	0	0	1	1
	2	0	0	1	1
	3	0	1	0	1
	4	0	1	0	1
	5	1	0	-1	-1
	6	1	0	1	1
	7	1	1	0	1
	8	1	1	0	1
	9	0	0	-1	-1
	10	1	0	1	1
	11	0	1	1	1
	12	1	1	1	1
	13	1	0	1	1
	14	1	0	1	1
	15	1	1	0	1
	N(1)	9	7	8	13
	N(0)	6	8	5	0
	N(-1)	0	0	2	2

Table A2.3.2: Quantitative Data for Carl, Day 2

	Turn	S1	S2	S3	S4
Game 1	1	0	0	-1	-1
	2	0	0	1	1
	3	0	0	1	1
	4	0	1	0	1
	5	0	1	0	1
	6	1	0	1	1
	7	0	0	1	1
	8	1	0	1	1
	9	1	0	1	1
	10	1	0	1	1
	11	0	1	1	1
	12	1	1	0	1
	13	1	1	1	1
	14	1	0	1	1
	15	1	0	0	1
	N(1)	8	5	10	14
	N(0)	7	10	4	0
	N(-1)	0	0	1	1
	Turn	S1	S2	S3	S4
Game 2	1	0	0	1	1
	2	0	1	0	1
	3	0	1	1	1
	4	0	1	1	1
	5	0	0	-1	-1
	6	0	0	1	1
	7	0	0	-1	-1
	8	1	0	1	1
	9	0	0	1	1
	10	0	0	1	1
	11	0	1	1	1
	12	1	0	1	1
	13	1	0	-1	-1
	14	0	1	1	1
	15	1	1	1	1
	N(1)	4	6	11	12
	N(0)	11	9	1	0
	N(-1)	0	0	3	3

Table A2.3.3: Quantitative Data for Carl, Day 3

	Turn	S1	S2	S3	S4
Game 1	1	0	1	1	1
	2	0	1	0	1
	3	0	0	1	1
	4	1	0	1	1
	5	1	0	1	1
	6	1	0	1	1
	7	0	1	1	1
	8	0	1	1	1
	9	1	1	1	1
	10	1	1	0	1
	11	1	1	1	1
	12	1	1	1	1
	13	1	1	1	1
	14	1	0	1	1
	15	1	1	1	1
	N(1)	10	10	13	15
	N(0)	5	5	2	0
	N(-1)	0	0	0	0
Game 2	Turn	S1	S2	S3	S4
	1	0	0	-1	-1
	2	0	-1	-1	-1
	3	1	-1	1	-1
	4	1	-1	1	-1
	5	0	1	0	1
	6	1	-1	-1	-1
	7	0	1	0	1
	8	0	1	1	1
	9	0	1	-1	-1
	10	1	1	1	1
	11	1	1	-1	-1
	12	1	1	1	1
	13	1	1	-1	-1
	14	1	1	1	1
	15	1	1	1	1
	N(1)	9	10	7	7
	N(0)	6	1	2	0
	N(-1)	0	4	6	8

Table A2.4.1: Quantitative Data for Dawn, Day 1

	Turn	S1	S2	S3	S4
Game 1	1	0	0	1	1
	2	1	0	-1	-1
	3	1	0	-1	-1
	4	0	0	-1	-1
	5	1	0	-1	-1
	6	0	1	1	1
	7	0	1	1	1
	8	0	0	-1	-1
	9	0	1	1	1
	10	1	1	0	1
	11	0	1	-1	-1
	12	-1	0	-1	-1
	13	0	1	-1	-1
	14	1	1	0	1
	15	1	0	-1	-1
	N(1)	6	7	4	6
	N(0)	8	8	2	0
	N(-1)	1	0	9	9
Game 2	Turn	S1	S2	S3	S4
	1	0	0	-1	-1
	2	0	0	-1	-1
	3	0	1	0	1
	4	0	1	0	1
	5	1	0	-1	-1
	6	1	1	0	1
	7	0	1	0	1
	8	1	1	0	1
	9	0	0	-1	-1
	10	1	0	-1	-1
	11	0	1	1	1
	12	0	1	-1	-1
	13	1	1	0	1
	14	1	1	0	1
	15	1	1	0	1
	N(1)	7	10	1	9
	N(0)	8	5	8	0
	N(-1)	0	0	6	6

Table A2.4.2: Quantitative Data for Dawn, Day 2

	Turn	S1	S2	S3	S4
Game 1	1	0	0	1	1
	2	1	0	-1	-1
	3	1	0	-1	-1
	4	0	0	-1	-1
	5	1	0	1	1
	6	0	1	0	1
	7	0	1	1	1
	8	0	0	1	1
	9	1	1	0	1
	10	1	1	0	1
	11	0	1	1	1
	12	1	0	1	1
	13	0	1	1	1
	14	1	0	1	1
	15	1	1	0	1
	N(1)	8	7	8	12
	N(0)	7	8	4	0
	N(-1)	0	0	3	3

	Turn	S1	S2	S3	S4
Game 2	1	0	0	-1	-1
	2	0	0	-1	-1
	3	1	1	0	1
	4	1	1	0	1
	5	1	0	-1	-1
	6	1	1	0	1
	7	0	-1	1	-1
	8	0	1	-1	-1
	9	0	0	1	1
	10	1	0	1	1
	11	0	1	1	1
	12	0	1	-1	-1
	13	0	1	1	1
	14	0	1	1	1
	15	1	0	-1	-1
	N(1)	6	8	6	8
	N(0)	9	6	3	0
	N(-1)	0	1	6	7

Table A2.4.3: Quantitative Data for Dawn, Day 3

	Turn	S1	S2	S3	S4
Game 1	1	0	0	-1	-1
	2	0	1	0	1
	3	1	0	-1	-1
	4	1	1	1	1
	5	1	0	-1	-1
	6	1	0	1	1
	7	1	1	-1	-1
	8	1	1	1	1
	9	1	1	1	1
	10	1	1	0	1
	11	1	1	0	1
	12	1	0	-1	-1
	13	1	1	1	1
	14	1	1	1	1
	15	1	1	1	1
	N(1)	13	10	7	10
	N(0)	2	5	3	0
	N(-1)	0	0	5	5

	Turn	S1	S2	S3	S4
Game 2	1	0	0	-1	-1
	2	0	1	1	1
	3	0	1	1	1
	4	0	1	1	1
	5	0	0	-1	-1
	6	1	0	-1	-1
	7	0	1	1	1
	8	1	1	0	1
	9	0	-1	-1	-1
	10	1	0	-1	-1
	11	1	1	1	1
	12	1	-1	1	-1
	13	1	1	-1	-1
	14	1	1	-1	-1
	15	0	1	-1	-1
	N(1)	7	9	6	6
	N(0)	8	4	1	0
	N(-1)	0	2	8	9

Table A2.5.1: Quantitative Data for Elsa, Day 1

	Turn	S1	S2	S3	S4
Game 1	1	0	0	1	1
	2	0	-1	-1	-1
	3	0	0	-1	-1
	4	0	-1	1	-1
	5	0	-1	1	-1
	6	0	1	0	1
	7	-1	-1	1	-1
	8	0	-1	-1	-1
	9	0	1	1	1
	10	1	1	0	1
	11	1	1	0	1
	12	1	0	1	1
	13	1	1	1	1
	14	0	1	1	1
	15	0	1	-1	-1
	N(1)	4	7	8	8
	N(0)	10	3	3	0
	N(-1)	1	5	4	7
Game 2	Turn	S1	S2	S3	S4
	1	0	0	-1	-1
	2	-1	0	-1	-1
	3	0	-1	-1	-1
	4	1	1	-1	-1
	5	0	1	-1	-1
	6	-1	-1	-1	-1
	7	-1	0	-1	-1
	8	0	1	-1	-1
	9	0	1	-1	-1
	10	0	1	0	1
	11	1	0	1	1
	12	1	1	1	1
	13	1	0	-1	-1
	14	1	1	-1	-1
	15	1	1	0	1
	N(1)	6	8	2	4
	N(0)	6	5	2	0
	N(-1)	3	2	11	11

Table A2.5.2: Quantitative Data for Elsa, Day 2

	Turn	S1	S2	S3	S4
Game 1	1	0	0	-1	-1
	2	0	1	0	1
	3	0	0	1	1
	4	1	1	0	1
	5	0	1	0	1
	6	1	0	1	1
	7	1	1	0	1
	8	1	1	1	1
	9	1	0	0	1
	10	0	1	0	1
	11	1	0	-1	-1
	12	1	0	1	1
	13	0	1	-1	-1
	14	0	1	-1	-1
	15	1	0	-1	-1
	N(1)	8	8	4	10
	N(0)	7	7	6	0
	N(-1)	0	0	5	5
Game 2	Turn	S1	S2	S3	S4
	1	0	0	1	1
	2	0	0	1	1
	3	0	1	1	1
	4	1	1	0	1
	5	0	1	-1	-1
	6	0	1	0	1
	7	0	0	1	1
	8	1	1	1	1
	9	-1	-1	1	-1
	10	0	1	1	1
	11	1	0	1	1
	12	1	1	-1	-1
	13	0	1	0	1
	14	1	1	0	1
	15	1	1	0	1
	N(1)	6	10	8	12
	N(0)	8	4	5	0
	N(-1)	1	1	2	3

Table A2.5.3: Quantitative Data for Elsa, Day 3

	Turn	S1	S2	S3	S4
Game 1	1	0	1	1	1
	2	0	0	1	1
	3	0	1	0	1
	4	0	1	0	1
	5	0	1	1	1
	6	0	0	1	1
	7	1	1	0	1
	8	0	0	1	1
	9	1	1	0	1
	10	1	1	0	1
	11	0	1	-1	-1
	12	1	1	-1	-1
	13	0	1	0	1
	14	1	1	1	1
	15	1	1	-1	-1
	N(1)	6	12	6	12
	N(0)	9	3	6	0
	N(-1)	0	0	3	3

	Turn	S1	S2	S3	S4
Game 2	1	0	0	-1	-1
	2	1	0	1	1
	3	0	1	1	1
	4	0	1	1	1
	5	1	0	-1	-1
	6	1	0	1	1
	7	0	1	1	1
	8	1	1	1	1
	9	1	1	-1	-1
	10	1	1	0	1
	11	1	1	0	1
	12	1	1	-1	-1
	13	0	1	1	1
	14	1	1	0	1
	15	1	1	-1	-1
	N(1)	10	11	7	10
	N(0)	5	4	3	0
	N(-1)	0	0	5	5

Table A2.6.1: Quantitative Data for Fred, Day 1

	Turn	S1	S2	S3	S4
Game 1	1	0	-1	1	-1
	2	0	-1	-1	-1
	3	0	-1	1	-1
	4	0	0	-1	-1
	5	0	-1	1	-1
	6	0	1	1	1
	7	0	1	0	1
	8	0	1	0	1
	9	1	1	1	1
	10	0	1	1	1
	11	-1	-1	1	-1
	12	1	1	0	1
	13	1	0	1	1
	14	1	0	-1	-1
	15	1	1	0	1
	N(1)	5	7	8	8
	N(0)	9	3	4	0
	N(-1)	1	5	3	7
Game 2	Turn	S1	S2	S3	S4
	1	0	0	1	1
	2	0	1	0	1
	3	0	-1	1	-1
	4	1	0	-1	-1
	5	0	0	1	1
	6	1	-1	-1	-1
	7	0	1	0	1
	8	0	1	0	1
	9	0	1	0	1
	10	1	1	0	1
	11	1	0	1	1
	12	0	1	0	1
	13	0	1	-1	-1
	14	0	1	-1	-1
	15	-1	0	-1	-1
	N(1)	4	8	4	9
	N(0)	10	5	6	0
	N(-1)	1	2	5	6

Table A2.6.2: Quantitative Data for Fred, Day 2

	Turn	S1	S2	S3	S4
Game 1	1	0	1	0	1
	2	0	1	0	1
	3	0	1	0	1
	4	0	0	-1	-1
	5	0	0	1	1
	6	1	1	0	1
	7	0	0	-1	-1
	8	1	1	0	1
	9	1	1	1	1
	10	0	1	-1	-1
	11	1	1	1	1
	12	1	1	0	1
	13	1	0	1	1
	14	1	1	0	1
	15	1	0	1	1
	N(1)	8	10	5	12
	N(0)	7	5	7	0
	N(-1)	0	0	3	3
	Turn	S1	S2	S3	S4
Game 2	1	0	0	-1	-1
	2	0	1	0	1
	3	0	-1	1	-1
	4	1	0	1	1
	5	0	0	1	1
	6	1	1	0	1
	7	0	-1	-1	-1
	8	0	0	1	1
	9	0	1	0	1
	10	1	1	0	1
	11	1	0	1	1
	12	0	1	1	1
	13	0	1	1	1
	14	0	1	1	1
	15	0	1	-1	-1
	N(1)	4	8	8	11
	N(0)	11	5	4	0
	N(-1)	0	2	3	4

Table A2.6.3: Quantitative Data for Fred, Day 3

	Turn	S1	S2	S3	S4
Game 1	1	0	0	-1	-1
	2	0	1	0	1
	3	0	-1	-1	-1
	4	1	1	-1	-1
	5	1	0	-1	-1
	6	1	0	1	1
	7	1	1	-1	-1
	8	1	1	1	1
	9	1	1	1	1
	10	1	1	0	1
	11	1	1	0	1
	12	1	0	1	1
	13	1	1	1	1
	14	1	1	1	1
	15	1	1	1	1
	N(1)	12	10	7	10
	N(0)	3	4	3	0
	N(-1)	0	1	5	5

	Turn	S1	S2	S3	S4
Game 2	1	0	0	-1	-1
	2	0	1	1	1
	3	0	1	1	1
	4	0	1	1	1
	5	0	0	-1	-1
	6	1	0	-1	-1
	7	0	1	1	1
	8	1	0	-1	-1
	9	0	1	0	1
	10	1	1	0	1
	11	1	1	1	1
	12	1	1	0	1
	13	1	1	0	1
	14	1	1	0	1
	15	1	1	1	1
	N(1)	8	11	6	11
	N(0)	7	4	5	0
	N(-1)	0	0	4	4

VITA

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